The Long and Short of Financial Development

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Abstract

Financial development enhances a producer's ability to raise capital to fund long term complex investments by improving the pledgeability of returns to financing households. Financial development should increase output and welfare. However, a general equilibrium analysis suggests this is not always so. We consider an economy where producers and consuming/financing households are distinct agents, where producers lack sufficient capital, and where households care about both pledgeable returns and liquidity. In this economy, the greater pledgeability of long-term project earnings can reduce long term production and overall welfare, even though it makes financing more accessible.Our results have implications for why economies face impediments to financial development and overall growth, especially when producer capital is scarce.

1 Introduction

A fundamental challenge in development is transitioning from simple, quick economic production processes with low returns to more complex, longer-term processes that generate higher returns. Financing such production is a complicating factor. To access household savings, producers must offer attractive claims with good returns. However, conflicts of interest, moral hazard, and low transparency can limit producers' ability to pledge future output from productive investments to households, especially for more complicated longer-run production processes. Financial development, for instance, through improved corporate governance, should increase the financeability of long-term complex projects by enhancing the pledgeability of returns. This, in turn, should increase the quantum of high-return production and foster development. Yet the impediments to financial development seem more than simply a lack of awareness of its benefits. What might they be?

We consider economic situations with three characteristics. First, only some agents are producers, and they have a choice of simple quick production or complex long production, and they often have limited capital. Therefore, production can be enhanced if producers can raise external funds by issuing financial claims to households. Second, while households often have sufficient surplus funds to invest, the pledgeability of producer output to financing households is typically low, lower still for long duration complex production than for short term production. This immediately implies that producers must co-invest with their own capital to make up the difference between required investment and available external funds. Consequently, the quantum of production will be limited by their capital. The combination of low producer capital and low relative pledgeability of long production also means that producers can only offer low rates of return to households, and the remaining return from production will accrue as rents to producers. These "rents from financing" accrue despite producers being competitive, and will play a critical role in the analysis. Third, financing households are also consumers (which is what we will call them from now on), and may have different and uncertain preferences for consumption over time. Their possible desire for early consumption, and hence liquidity, will affect their allocations to and pricing of financial claims. All three elements are critical to our interesting results.

Let us be more specific. Competitive and homogeneous producers can undertake either short-term lower-return investment (such as planting seeds to produce fresh vegetables, or holding inventories of commodities to trade them) or long-term higher-return complex investment (such as that needed to produce canned tomato paste). Producers value consumption at any time equally, and so only care about their overall returns.

Each of these investments has associated with it a degree of pledgeability — defined as the share of output it produces that can be committed to be paid to outside investors. So short pledgeability is the degree to which the output from the short term investment can be paid out. For inventory investment, think of more effective and easily monitored warehousing technology that ensures the pledged inventory is available to support any lender's efforts to collect promised payment. Similarly, long pledgeability reflects the quality of corporate governance, which ensures the long term investment is managed in the interests of investors.

Producers are endowed with some capital but can also secure funding for a portion of their real investments by issuing financial claims to consumers. The amount of funding they can obtain is limited to the pledgeable value of their production output.

Consumers in our model also have some capital but cannot produce on their own. For simplicity, we also assume they do not have independent avenues to save on their own, though access to low return storage is easily accommodated. They are also uncertain about the date on which they need to consume. Therefore, they will value the liquidity of financial claims, defined as the return they can obtain at an early date, in addition to valuing long-term returns.

For most of our analysis, we assume that there is a competitive financial market on each date. This market allows competing producers to issue financial claims to consumers initially and later allows consumers to trade financial claims with each other. Importantly, limited producer capital coupled with limited pledgeability of output to consumers gives producers rents from financing that cannot be competed away. These rents may differ for short and long assets.

Competition between competitive producers (all with access to the same technologies) requires them to pass through to consumers as much of the output produced as is pledgeable. Because producers can undertake either short or long term investment and can raise funding in a competitive market, producer returns on either investment, including the rents from financing, must be equal if both investments are undertaken; else, only the production with the higher return to producers will be undertaken. The rates of return available to consumers on short term and long term financial claims depend on the degree of pledgeability of output from each maturity as well as on the market price consumers pay for those claims when issued or resold. If long-term claims resell at interim dates for low prices (offering high returns to buyers), long-term claims are illiquid.

The core of our analysis focuses on a key conflict of interest: when an investment becomes more pledgeable, producers can commit to pay out more of that asset's output to consumers, and competition forces them to do so. However, this affects the producer's rents from financing that asset, and consequently the attractiveness of producing more of it. Consumer returns from buying financial claims on the asset move in the opposite direction to producer returns, which also affects their allocations. This implies that an increase in the pledgeability of the long asset, which we term *financial development*, does not always increase producer production or consumer financing of it, unlike what a partial-equilibrium analysis might suggest.

Some examples may help fix ideas. Start with the case when assets are fully pledgeable. In that case, competitive producers will pledge all the returns from externally-financed projects to consumers (so they get no rents from financing), and the producers do not need to make up financing shortfalls in any asset with their own capital. They will invest their own capital in higher return long production for their own consumption. Consumers allocate their capital by trading off the higher return from long-term claims and the liquidity offered by short-term claims.

Now consider lower levels of asset pledgeability. Start first with the case where producers have large amounts of capital relative to consumers, and so can co-invest as needed. Producers pay out only the pledgeable portion of output, but they need raise only a small fraction of the investment needed in each project from consumers, co-investing the rest. Producer competition will ensure that the rents from financing the long asset are driven to zero, and consumers are paid a return as if the long asset were fully pledgeable. Consumers will get higher returns from the long claim, with the liquidity benefits from the short financial claim motivating them to hold both claims in equilibrium.

By way of contrast, consider the case where producers have no capital. In that case, the output that will accrue to consumers is only what is pledgeable. Since the consumer has to put up all the funds for investment, she might allocate them to financing only the short asset if the pledgeable returns from the short asset exceed the pledgeable returns on the long asset. In this case, pledgeability determines what is produced, and the lower pledgeability of the long asset may cause it to be dominated. However, the producers make substantial "rents from financing" since they pay out only the pledgeable part of any output, retaining the rest of the output for themselves despite not investing a cent, and despite markets being competitive. The rents stem from the producers' monopoly over production, with the lack of pledgeability (and of producer capital) effectively limiting competition. The paper focuses on what happens when neither financial development or capital are at extremes.

We will see that the level of financial development affects how changes in financial development play out. A critical level is when the pledgeable returns of the high return long term project just equal the pledgeable returns on the more pledgeable low return short term project. *Ceteris paribus*, above this level of financial development, project returns and project pledgeability are *aligned*, that is, higher return projects generate more pledgeable output, while they are *misaligned* at levels below, in that the lower return project generates more pledgeable output.

At really low levels of financial development (and low producer capital), returns and pledgeability are grossly misaligned, and only the short term project will be undertaken. Improvements in financial development over a range have no effect on project choice or output. The outcomes here are reminiscent of primitive economies where the accent is on simple subsistence production and commodity trade.

At higher levels of financial development, while returns and pledgeability are still misaligned, we will see financial development helps increase producer and consumer allocations to the long asset. However, producers get a disproportionate share of the additional returns, so much so that consumers are worse off. So in this region, consumers would not support financial development.

Matters change considerable when financial development increases further, aligning returns with pledgeability, so that higher return projects also have more pledgeable output. Intriguingly, at these levels of financial development, consumers' liquidity concerns ensure their capital allocations to different financial claims are fixed. So an increase in the pledgeability of the long asset, that is, an increase in financial development, shows up in a lower consumer price for the long asset, higher consumer returns, and thus lower rents from financing to the producer. Producers will have incentives to tilt towards production that is less pledgeable, that is the short term lower return asset, which contradicts the partial-equilibrium intuition that an increase in pledgeability of an asset, and thus an increase in the financing available for it, should increase its production. Over a range of financial development, any increase reduces the share of aggregate capital that is devoted to long projects, and reduces producer welfare, as well as overall output, while enhancing consumer returns. Consequently, producers have an incentive to oppose further financial development in this region, akin to a middle-development trap.

Finally, at very high levels of long pledgeability, that is, financial development, the elimination of rents from financing longs will make producers more open to further increases in pledgeability. Thus, at very high levels of financial development, the conflicts of interest over greater pledgeability dissipate. Of course, as we discussed earlier, if producers have sufficient own capital relative to consumers (that is, producers are sufficiently wealthy) so as to reduce the need for external financing significantly, rents due to financing will not be material even at moderate levels of financial development, and conflicts of interest over financial development will be small. A whole range of interesting possibilities, and sharing of returns and rents, as well as conflicts over financial development, exist for intermediate cases. We obtain a political economy of financial development which carries the sobering message that conflicts of interest over further development dissipate only when financial development is already at a high level or when producers are wealthy. This suggests a version of what is termed the Matthew effect ("to every one who has will more be given,...") may apply to financial development also.

We also examine the effects of increases in short asset pledgeability, what might be termed *credit development*. We find that it makes the consumer better off, and make the producer (weakly) worse off. The effects on overall welfare are, once again, more ambiguous.

Research has typically focused on the returns available to consumers as the pledgeability of all financial claims increases (see, for example, Holmström and Tirole (1998), Kiyotaki and Moore (1997), and He and Krishnamurthy (2013)), but not on the resulting equilibrium incentives for producers to allocate resources to the production of different assets, the rents they collect, and their incentives to push for further financial development.[1] There is much to learn from examining these. Earlier work in a similar vein to ours include Ebrahimy (2022) and Matsuyama (2007). They examine the effects of differing pledgeability of assets of the same maturity and its impact on the efficiency of investment. Their models do not incorporate consumers who care about liquidity or account for possible future trading. Our model includes assets with different maturity and consumers with uncertain needs for future liquidity , which leads to our interesting novel results, as we will explain later. Finally, our paper also relates to work on the political economy of financial development (see, for example, Rajan and Zingales (2003)).

The rest of the paper is as follows. In section 2, we present the model, and analyze equilibria for various parameters in section 3. In section 4, we examine incentives for financial development given the comparative statics of various equilibria if decision making is in different hands, and relate our work to the literature. In section 6 we examine the actions that a social planner would take under different constraints. We conclude in section 7.

2 Model

2.1 Agents and Preferences

Consider an economy with three dates $t = 0, 1, 2$, and two categories of agents, consumers and producers.

There are a continuum of consumers of total measure λ. Each consumer is endowed with with one unit of capital at $t = 0$. With *i.i.d.* probability $1 - q$, a consumer turns out to be early; with probability q, he turns out to be late. An early consumer only cares about

[1]There is also a large literature on the role of producer wealth when there are financial constraints, including Bernanke and Gertler (1989) and Hart and Moore (1994).

consumption at $t = 1$, so his utility function is C_1, whereas a late consumer's utility function is $C_1 + C_2$. Consumer type (early or late) is private information of each consumer. For now, we assume consumers preferences are linear and thus risk-neutral. Other than this linearity, these preferences are identical to those in Diamond and Dybvig (1983). The linearity is for simplicity and most of our results, such as resource allocation and equilibrium prices, remain unchanged if consumers are risk averse. Producers are of measure one and each is endowed with one unit of capital.[2] They can consume at both $t = 1$ and $t = 2$, and their payoff is $\Pi_1 + \Pi_2$ where Π_t is their payoff at date t.

2.2 Asset and Pledgeabilities

Producers can invest in two types of real assets (using their capital and the funding raised by issuing financial claims to consumers) at date 0. Both assets are constant returns to scale investments available to all producers, but only to them. One is a short-term asset (henceforth short asset) with a return per unit invested of $R \geq 1$ at $t = 1$. The output of this investment should be thought of as a tradeable consumption good. The second asset is a long-term one (henceforth long asset) with a return of $X > R$ at $t = 2$ but zero return if liquidated early at $t = 1$. This asset could be thought of as a sophisticated asset, that is, a project or firm that pays off in the long run.

Producer investments are made with the producers' own capital as well as the resources they raise from consumers. Not all of an asset's return can be paid out to consumers. In the case of the short asset, the producer may need to retain some "skin in the game" upfront to assure buyers of claims on it that they will get their share of output. This is especially the case if the production process requires effort. An alternative interpretation is that there are defects in the production process, implying that only a fraction of the short asset's output is consumable or exchangeable by consumers, while the rest can only be consumed by the producer (think of the producer producing misshapen or unattractive vegetables that are intrinsically edible but are unacceptable to consumers because they are uncertain about quality). We do not differentiate between these different microfoundations and assume that only a fraction γ_S of the short-term asset's output is payable to consumers. We refer to γ_S as *short pledgeability*. Institutional developments such as better banks, more reliable warehouses where inventory can be stored and monitored, better enforcement of collateral pledges, etc., would all contribute to higher short pledgeability.

Similarly, we assume only a fraction γ_L of the long-term asset's output at $t = 2$ is pledgeable, where γ_L *is long pledgeability*. The reasons only a portion is pledgeable could

[2]The assumption that producers each have one unit of capital is made without loss of generality, because their size (1) is different from consumers (λ).

be similar to those for the short asset. In addition, though, long assets require greater probity of, and incentives for, the producer since she has more time and cover (because of the more complex nature of the asset) to steal output, or shirk. In that sense, long pledgeability proxies for the governance oversight over the long term asset. Improvements in accounting standards, corporate disclosure and transparency, corporate governance, etc., would all contribute to long pledgeability,

We will use the term long pledgeability interchangeably with *financial development*. We will associate short pledgeability with *credit development*.

If either short or long pledgeability were zero, no financial claim of that maturity could be issued and only producer capital could finance that investment, and consumer capital would be unusable. To address such possibilities, and at the cost of notational complexity, we could easily accommodate the availability of a low-return own investment opportunity as an outside option for consumers.

For now, we assume both production technologies are only available at $t = 0$. In other words, there is no other means for consumers to save from $t = 1$ to $t = 2$. However, our assumption that late consumers value consumption on both date 1 and 2 ($=C_1 + C_2$) is equivalent to having them value only date 2 consumption, while being able to store pledgeable consumption goods between those dates at a zero rate of return.

2.3 Financial Market and Rates of Return

Markets open at $t = 0$ and $t = 1$. In the $t = 0$ financial market, the producer can sell financial claims against the pledgeable output produced by the real assets. The financial assets will offer rates of return between date 0 and future dates and can be traded at date 1.

Specifically, let p_L be the quantity of date-0 capital consumers contribute to buy a financial claim written against one unit of investment in the long asset, which delivers cash flows $\gamma_L X$ at $t = 2$ to the consumer. This is the date-0 price of the long claim. Let p_S be the price of a claim written on one unit of the short asset, delivering cash flows $\gamma_S R$ at $t = 1$. If $p_L<1$, a long claim is produced with a fraction p_L of consumer capital and $1 - p_L$ of producer capital, while for $p_S < 1$, a short claim is produced with a fraction p_S of consumer capital and $1 - p_S$ of producer capital. Any assets produced that do not back claims are both retained and funded by producers.

Some additional notation will be useful. Let consumers investing at t receive promised rates of return, $r^a_{t\tau}$, between dates t and τ for asset $a \in \{S, L\}$. So $r^L_{02} = \dfrac{\gamma_L X}{p_L}$ and $r^S_{01} = \dfrac{\gamma_S R}{p_S}$, respectively for the long and short claim.

Once the uncertainty on when they will consume is resolved, some consumers will have gains from trading in the $t = 1$ financial market, where only consumers can trade. Let b_F be the endogenous date-1 price per unit of a long financial claim (that is, a claim on $\gamma_L X$). Trading the long claim at this price on date 1 provides a rate of return between dates 1 and 2 of $r^L_{12} = \frac{\gamma_L X}{b_F}$. Clearly, only late consumers want to buy the claim. If so, the price b_F cannot be so high that the late consumer prefers consuming b_F immediately at date 1 rather than waiting till date 2 and consuming $\gamma_L X$. Therefore, $b_F \leq \gamma_L X$, otherwise, late consumers will not buy at $t = 1$. Put differently, the second period gross return on the long financial claim, r^l_{12}, cannot go below 1.

The role of a short-term financial claim is two-fold. First, it offers cash flows for consumption when consumers are early types. Second, when they are late, it offers cash flows for them to buy long-term financial claims or to use for immediate consumption. The ability to buy is particularly valuable when long-term claims are illiquid, selling at discounted interim date prices (that is, $b_F < \gamma_L X$) which allow the date-1 buyers to enjoy higher returns $r^L_{12} > 1$. The more consumers are induced to invest in the long claim relative to the short claim at date 0, the greater the interim-date discount, which imposes a natural constraint on the attractiveness of the long claim, offsetting the return on the underlying asset, X, and its pledgeability. Naturally, consumers will demand the long-term claim only if it offers a sufficiently high return, taking into account the potential need to trade it at a discounted price.

2.4 Equilibrium Definition and Preliminary Analysis

Let the representative consumer invest share θ and $1 - \theta$ of their capital λ at date 0 in long claims and short claims respectively. A representative producer has 1 unit of capital and allocates y_L to the production of the partly externally financed long asset(backing the long claim), y_S to producing the short asset (backing the short claim), and $1 - y_L - y_S$ to long asset production that she self-finances entirely for personal consumption of its return. Consumers will buy all of the financial assets produced. The producer never entirely self-finances any short production, because long investments are more productive, $X > R$, and she values cash flows equally at both $t = 1$ and $t = 2$. Then the economy is characterized by six unknowns $\{\theta, y_L, y_S, p_S, p_L, b_F\}$.

A producer's payoff then is

$$\Pi = \max_{y_L, y_S} \underbrace{\frac{y_S}{1 - p_S}}_{\text{short production}} \underbrace{(1 - \gamma_S) R}_{\text{non-pledgeable short}} + \underbrace{\frac{y_L}{1 - p_L}}_{\text{long production}} \underbrace{(1 - \gamma_L) X}_{\text{non-pledgeable long}} + (1 - y_L - y_S) X.$$

Note that due to producer competition neither p_L nor p_S can be greater than 1 for that would mean the consumer entirely finances investment and more, so every producer would compete the relevant price down to 1, given they have no personal cost of production. It is clear that the producer does not self finance long production for own consumption (the last term) when $\frac{(1-\gamma_L)}{(1-p_L)} > 1$ or equivalently $p_L > \gamma_L$, since it is then more profitable for the producer to finance externally using $(1-p_L)$ of own resources and p_L of the consumer's funds, thereby obtaining fraction $(1-\gamma_L)$ $(> (1-p_L))$ of the output.

When producers issue both long and short financial claims, they must earn the same rate of return on investing producer capital in either asset for the purpose of creating financial assets. This incentive for investing in real assets for this purpose leads to the following first order condition.

$$\frac{(1-\gamma_S)\,R}{1-p_S} = \frac{(1-\gamma_L)\,X}{1-p_L}. \tag{1}$$

This implies that $1 - p_L = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-p_S)$, and that the fraction of producer capital contributed to producing each unit of long claim is a fraction $\frac{(1-\gamma_L)X}{(1-\gamma_S)R}$of that contributed to each unit of short claim. Intuitively, higher γ_L reduces the contribution of producers per unit of long relative to their contribution per unit of short, while higher γ_S has the reverse effect – relative producer capital allocation moves in the opposite direction to relative pledgeability.

Also note that the rent the producer obtains from financing the long asset is

$$\frac{y_L\,(1-\gamma_L)\,X}{1-p_L} - y_L X = \frac{y_L X\,(p_L-\gamma_L)}{1-p_L}. \tag{2}$$

So the rent from financing comes from the producer's ability to sell γ_L of financial claims on the long asset for $p_L > \gamma_L$, and similarly for the short asset. These rents will be critical in understanding incentives for development.

The consumer demand for financial claims depends on the return they can achieve from the pledgeable returns they deliver. The expected payoff of the consumer is

$$U = \max_{\theta}\ (1-q)\left(\underbrace{\frac{\theta}{p_L} b_F}_{\text{sell long-financial}} + \underbrace{\frac{1-\theta}{p_S}\gamma_S R}_{\text{dividend short-financial}} \right) + q\left(\underbrace{\frac{\theta}{p_L}\gamma_L X}_{\text{dividend long-financial}} + \underbrace{\frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X}_{\text{buy long-financial using dividend from short-financial}} \right)$$

The first term in large parentheses is the payoff conditional on turning out to be an early consumer. In it, the first term is the value from selling holdings of the long financial claim and consuming the proceeds, the second is the value of consuming the payoff from holdings of the short financial claim. The terms within the second set of large parentheses is the payoff conditional on turning out to be a late consumer. In it, the first term is the value of consuming the payoff from the long financial claim, the second is the value from buying more of the long financial claim using the payoffs from the short financial claim. When consumers hold both assets, the F.O.C. w.r.t. θ implies that the consumer's expected returns (given the distribution of their liquidity shocks) are equalized across both long and short financial claims.

$$(1-q)\,\frac{b_F}{p_L} + q\frac{\gamma_L X}{p_L} = (1-q)\,\frac{\gamma_S R}{p_S} + q\frac{\gamma_S R}{p_S b_F}\gamma_L X. \tag{3}$$

Market clearing at $t = 0$ requires

$$\underbrace{\lambda\frac{\theta}{p_L}}_{\text{demand for long financial}} = \underbrace{\frac{y_L}{1-p_L}}_{\text{supply of long financial}} \tag{4}$$

$$\underbrace{\lambda\frac{1-\theta}{p_S}}_{\text{demand for short financial}} = \underbrace{\frac{y_S}{1-p_S}}_{\text{supply of short financial}}. \tag{5}$$

From the market clearing conditions (4) and (5) above, we can easily derive the prices:

$$p_L = \frac{\theta\lambda}{\theta\lambda + y_L} = \frac{1}{1+\frac{y_L}{\lambda\theta}} \tag{6}$$

$$p_S = \frac{\lambda(1-\theta)}{\lambda(1-\theta)+y_s} = \frac{1}{1+\dfrac{y_s}{\lambda(1-\theta)}}. \tag{7}$$

These expressions are intuitive. If the producers puts capital of y_L into long production and consumers put in $\lambda\theta$, the date 2 pledgeable payment to consumers is $(\theta\lambda + y_L)\gamma_L X$ and the consumer rate of return on longs can be as high as $\frac{(\theta\lambda+y_L)\gamma_L X}{\theta\lambda}$. Competition among producers push the consumer's rates of return on financial claims to their upper bounds, determined by the relative amount of producer and consumer capital invested in a given asset. At this upper bound, this must equal $\dfrac{\gamma_L X}{p_L}$, so the date-0 price of pledgeable payoffs of $\gamma_L X$ is then $p_L = \frac{\theta\lambda}{\theta\lambda+y_L}$. Following similar logic, $p_S = \frac{\lambda(1-\theta)}{\lambda(1-\theta)+y_S}$. Note that higher the producer allocation to an asset relative to consumer allocation, lower the claim price, and higher the consumer return. Hence, much of the comparative statics analysis will involve tracing how the allocations move.

At the $t = 1$ financial market, late consumers (a total of q) want to buy the long asset; early consumers (a total of $1 - q$) want to sell. Market clearing implies

$$b_F = \min \left\{ \frac{q \frac{1-\theta}{p_S} \gamma_S R}{(1-q) \frac{\theta}{p_L}}, \gamma_L X \right\}, \tag{8}$$

where the price is capped when the quantum of long assets coming on the market at date 1 relative to the purchasing power of all potential buyers is low.

Equations (1)-(8) solve the model. We also define overall welfare as the simple sum of the payoff to the consumers and producers, i.e., $\lambda U + \Pi$. The full welfare analysis is in the appendix and described in section 6.3.

Before proceeding with the full solution, let us discuss some preliminary results.

Lemma 1. *When* $b_F = \frac{q \frac{1-\theta}{p_S} \gamma_S R}{(1-q) \frac{\theta}{p_L}}$, *then* $\theta = q$.

A proof is straightforward by plugging $b_F = \frac{q \frac{1-\theta}{p_S} \gamma_S R}{(1-q) \frac{\theta}{p_L}}$ into (3) .

This result says if the date 1 price of the long asset is set to clear the market where early consumers sell all of their long assets to late consumers for all of their short assets, and the consumer's FOC holds with equality, it must be that they allocate exactly a fraction q of their capital to the long asset at date 0. The demand for a financial claim must account for both the return from consuming its return and using respectively short claims to buy other longs or selling long claims for proceeds from shorts in the future. This is a no arbitrage condition for consumer investment and is similar to that in Jacklin (1987). Intuitively, if consumers allocate more than q to long at $t = 0$, the price of the long asset at the $t = 1$ market is too low to satisfy (3) , inducing them to allocate away from long, and vice versa.

Lemma 2. *In equilibrium,* $r^S_{01} = r^L_{01}$ *if* $\theta \in (0, 1)$, *where* $r^S_{01} = \dfrac{\gamma_S R}{p_S}$ *and* $r^L_{01} = \frac{b_F}{p_L}$. *If* $b_F = \gamma_L X$, *then* $\frac{\gamma_L X}{p_L} = \frac{\gamma_S R}{p_S}$ *so* $r^L_{02} = r^S_{01}$.

A proof is straightforward from Equation (3) by simply rewriting the equation as

$$(1-q) \left(\frac{b_F}{p_L} - \frac{\gamma_S R}{p_S} \right) = q \left(\frac{\gamma_S R}{p_S} - \frac{b_F}{p_L} \right) \frac{\gamma_L X}{p_L} \frac{p_L}{b_F},$$

which can be rewritten in terms of returns

$$(1-q) \left(r^L_{01} - r^S_{01} \right) = q \left(r^S_{01} - r^L_{01} \right) r^L_{12}.$$

This implies that as long as consumer's allocation is interior, i.e., $\theta \in (0, 1)$, the returns between $t = 0$ and $t = 1$ offered by the short and long financial claims are identical. As a result, the early consumer can trade out of the long claims he has at date 1 and receive a value of the short claim as if he had invested in the short claim all along. Similarly, the late consumer can sell the short claims he has and buy the long claims so he receives the value he would have if he had invested up front in the long claim. Put differently, the interim price is set at precisely the level that long payoffs are converted to short payoffs and vice versa so that the consumer's holding does not matter, given prices. The ability to trade once again eliminates the risk to the consumer from holding the wrong asset, given his type. Furthermore, in the particular case where $b_F = \gamma_L X$, not only do the short and long asset deliver the exact same return on date 1, the date 1 to 2 return on the long asset is $\frac{\gamma_L X}{b_F} = 1$, that is, the long financial claim is liquid. There are no essential differences in return between the two assets.

Lemma 3. *In any equilibrium, it cannot be that long dominates short for consumers, i.e., $\theta = 1$ is not possible.*

We can show this result by contradiction. If $\theta = 1$, $b_F \to 0$ since there is no purchasing power to pick up the longs that early consumers want to sell, presenting an astronomical return to any late consumer holding short claims. So it cannot be that no short claims are issued at date 0.

2.5 Simple Benchmarks

Let us start with some simple cases where only the preferences and trading opportunities of consumers are relevant and the available rates of returns to consumers depend only on technology and competition. In the first benchmark, the cash flows are fully pledgeable, so that the producers do not need to make up capital shortfalls in any financial claim (they will invest their capital in long production for own consumption). In the second benchmark, even though cash flows are partially pledgeable, producers have no capital to deploy. We will see that in both cases, the financial claims are funded entirely by the consumer.

Full pledgeability, $\gamma_L = \gamma_S = 1$

Full pledgeability combined with competitive producers with constant returns to scale investments immediately implies that all of the return from consumer investment must accrue to consumers. That is, the zero excess profit condition for producers immediately implies that $r_{01}^S = R$ and $r_{02}^L = X$ and $p_L = p_S = 1$. The producers invest their own capital only to finance their own consumption, investing it in long assets.

The only endogenous choice is the consumer allocation of initial capital given $r_{01}^S = R$ and $r_{02}^L = X$. The rate of return from date 1 to 2 in the market is r_{12}^L and it comes from the option to sell longs worth X for $b_F = \frac{X}{r_{12}^L}$ at date 1 or to buy longs at date 1 and earn r_{12}^L at date 2. The first order condition for an interior optimum when both assets are held is:

$$(1-q)\, b_F + qX = (1-q)\, R + q\frac{R}{b_F}X,$$

which has a unique solution $b_F = R$. Any other solution would lead one asset to be dominated for the consumer.

Market clearing at date 1 then requires:

$$b_F = \min\left\{\frac{q(1-\theta)R}{(1-q)\,\theta}, X\right\}.$$

If both assets are held, then whenever $b_F < X$, $b_F = R$, which means the initial consumer allocation is $\theta = q$. This is always the outcome because $X > R$.[3]

Producers have no capital (implying $\lambda \to \infty$).

Turn next to the case where producers have no capital of their own and there is incomplete pledgeability. It must be that consumers provide all the capital for assets and thus for claims, and thus $p_S = p_L = 1$. The returns offered to consumers would need to be $r_{01}^S = \gamma_S R$ and $r_{02}^L = \gamma_L X$, leaving unavoidable rents to producers. The first-order condition for both assets being held and interim price become

$$(1-q)\, b_F + q\gamma_L X = (1-q)\, \gamma_S R + q\frac{\gamma_S R}{b_F}\gamma_L X$$

$$b_F = \min\left\{\frac{q(1-\theta)\gamma_S R}{(1-q)\,\theta}, \gamma_L X\right\}$$

Interestingly, the discussion in the full-pledgeability case continues to hold with these rates of return. The only difference is that it is possible that the pledgeable return on shorts exceeds that on longs, or $\gamma_S R \geq \gamma_L X$. In this case, when the inequality is strict, shorts dominate longs as a consumer investment, implying $\theta = 0$ and the shadow price of any long financial asset at date 1 is $b_F = \gamma_L X$ (no trade actually takes place, since no long assets are produced).

As in the case of complete pledgeability, the rates of return offered to consumers are set directly by technology and competition. The commonality is that producer capital is not in play. When there is incomplete pledgeability and producers have some capital, they

[3] It is easily derived that given $X > R$, the solution to the first-order condition for consumers cannot be at a corner: if all invest in long, then $\theta = 1$, $b_F = 0$, inducing consumers to allocate to short; if all invest in short, then $\theta = 0$ and $b_F = X$, inducing consumers to allocate to long.

would want to compete for consumer funding by investing some of their own capital to offer consumers a higher pledgeable return for a given consumer investment. In this case, the incentives of consumers and producers interact to determine the returns available on financial assets. Recall that if producers wish to get consumers to hold any long financial claims, these claims would need to offer at least as high a return to maturity as short claims. We now turn to this more general case.

3 Decentralized Market Equilibrium Outcome

3.1 Equilibrium Regimes, Aligned and misaligned returns

Limited pledgeability of an asset has two important effects. First, lower pledgeability of an asset reduces the rate of return that a claim on it offers to consumers for a given allocation of capital by producers and consumers (recall from equations (6) and (7) that the allocations fully determine the price of these financial claims). Second, lower pledgeability usually increases the rate of return an asset offers to producers, because producers retain the non-pledgeable part of output, and do not compete down the returns in selling financial claims to consumers as much. Other things equal, reduced pledgeability reduces consumer demand for a given financial claim and increases producer willingness to supply it. The aggregate quantum of consumer capital relative to producer capital, λ, also makes a difference. The scarcer the producer capital, the more producer rents, distorting returns to consumers and altering production decisions. Finally, there is an interesting interaction between investment in short and long assets because the payoffs on short financial claims fund the purchases of long financial claims resold in the secondary market at date 1.

We have argued that short pledgeability is naturally likely to be higher than long pledgeability. In institutionally underdeveloped economies, it is possible that financial development be so low that $\gamma_S R > \gamma_L X$. In such a situation, pledgeable returns are *misaligned* with underlying asset returns. Less productive assets are more pledgeable. Of course, at high levels of financial development, *ceteris paribus* , eventually $\gamma_S R \leq \gamma_L X$, and pledgeable returns and underlying returns will be *aligned*.

The related literature (see, for example, Ebrahimy (2022) and Matsuyama (2007)) has focused on the case of misaligned returns. We also find interesting results when returns are aligned and more profitable long term investments are easier to finance, as in emerging or developed economies.

We now discuss the following possible (and exhaustive) cases, holding the pledgeability of the short asset constant at $0 < \gamma_s < 1$, and varying the pledgeability of the long asset.

1. *Short dominance :* At very low levels of γ_L, producers will find inadequate financing for the long asset, and will find the returns from investing in it dominated by investing solely in the short asset.

2. *Short glut:* When γ_L increases sufficiently,producers will see their return on the production of long assets rise to their return on the production of short assets and a small fraction of long assets and financial claims will start getting produced. There will be a glut of short claims relative to long, ensuring the scarce long financial claim will be liquid in that it sells for full face value at date 1.

3. *Illiquid long* : When γ_L increases further, and sufficient producer and consumer capital shifts to long production, long financial claims have an interim price b_F less than $\gamma_L X$, and hence are illiquid.

4. *No long rent*: When γ_L is higher still, the date-0 price of the long financial claim is driven down to the point that producers offer consumers the full rate of return available from long production and there are no rents associated with externally financed long (or short) production.

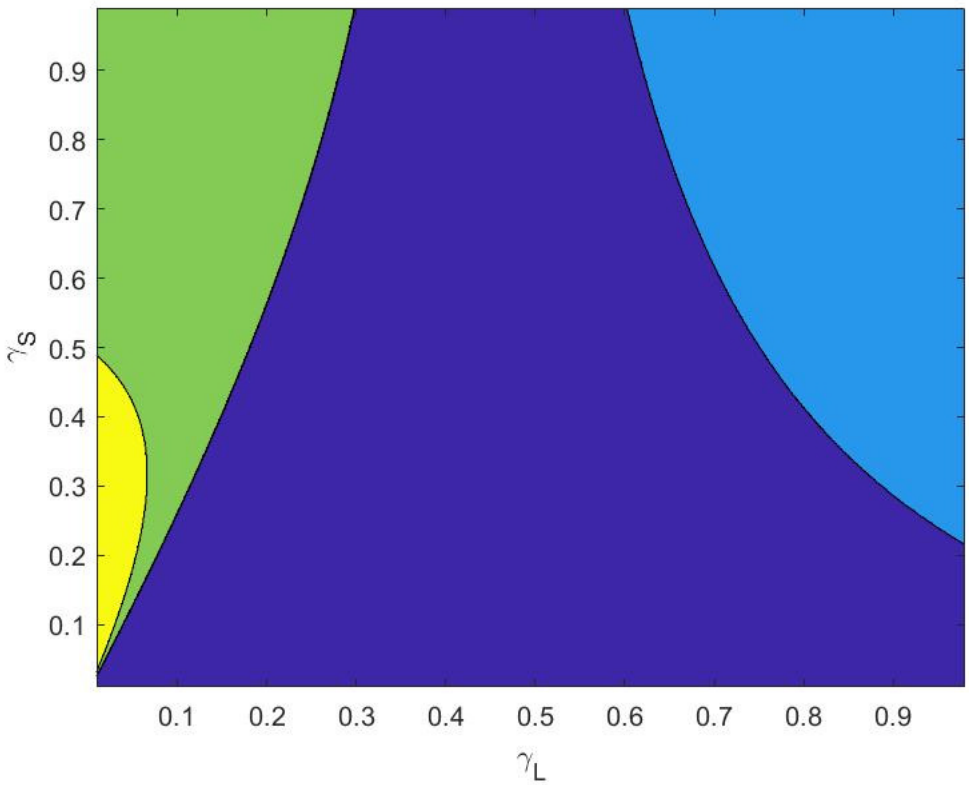
The first two cases, short dominance and and short glut cannot occur when returns are aligned with pledgeability ($\gamma_L X > \gamma_S R$), as we show shortly in 2. Conversely, all four cases are possible when returns are misaligned ($\gamma_S R > \gamma_L X$).

Proposition 1. *There exists a unique equilibrium.*

Proof. See Appendix □

Figure 1 anticipates our general results on the effects of financial development.

Figure 1: Equilibrium Cases a function of γ_L and γ_S



This graph illustrates the equilibrium regions as a function of γ_L and γ_S. The yellow region is where the short asset dominates; the green region is where there is a relative glut of the short asset so that the long asset is liquid, the dark blue region is where the long asset is illiquid but also enjoys a rent from financing, and the light blue region is where the long asset is illiquid but there is no rent from financing.

3.2 Variation in the pledgeability of long assets

3.2.1 Short dominance

If $\gamma_L \leq \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}$, given the shadow prices it is unprofitable for the producer to produce the long asset or the consumer to invest in the associated financial claim. In such an equilibrium, $y_L = 0$, and $\theta = 0$. All of consumer capital, λ, goes into short claims. We will show the producer will not retain long assets so all her resources are devoted to producing the short asset and $y_S = 1$. If so, $p_S = \frac{\lambda}{\lambda+1}$. The producer must prefer producing short assets to producing and retaining long so $\frac{(1-\gamma_S)R}{1-p_S} \geq X \Rightarrow (\lambda+1)(1-\gamma_S)R \geq X$. When all assets are short, any early consumer who deviated and had a long to sell would obtain the full date 2 value $b_F = \gamma_L X$ from a late buyer with plenty of purchasing power. That is, the shadow $b_F = \min\left\{\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}, \gamma_L X\right\} = \gamma_L X$[4] In this case, the consumer will hold only the short financial claim requiring that it offer as high a return as a long claim, implying a consumer reservation price, $\underline{p}_L$, for longs:

$$(1-q)\frac{\gamma_L X}{p_L} + q\frac{\gamma_L X}{p_L} \leq (1-q)\frac{\gamma_S R}{p_S} + q\frac{\gamma_S R}{p_S}$$
$$\Rightarrow \frac{\gamma_L X}{p_L} \leq \frac{\gamma_S R}{p_S} \Rightarrow p_L \geq \underline{p}_L \equiv \frac{\gamma_L X}{\gamma_S R} p_S = \frac{\gamma_L X}{\gamma_S R}\frac{\lambda}{\lambda+1}$$

Put differently, for consumers to shun long claims paying $\gamma_L X$, the fraction of their own capital that needs to go into each unit of long must be so high as to depress the returns below what they can get from investing in shorts.

Finally, it must be that the producer finds it less profitable to produce the long claim instead of the short, so

$$\frac{(1-\gamma_L)X}{1-p_L} \leq \frac{(1-\gamma_S)R}{1-p_S} \Rightarrow 1-p_L \geq \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-p_S) \Rightarrow p_L \leq \bar{p}_L \equiv 1 - \frac{(1-\gamma_L)X}{(1-\gamma_S)R}\frac{1}{\lambda+1}.$$

Put differently, the rent from financing available from producing longs per unit of producer capital that must be deployed is swamped by the rent available on shorts.

The set of p_L satisfying both constraints for no long claims to be held or long assets produced, is non-empty if

$$\underline{p}_L \leq \bar{p}_L \Rightarrow \frac{\gamma_L X}{\gamma_S R}\frac{\lambda}{\lambda+1} \leq 1 - \frac{(1-\gamma_L)X}{(1-\gamma_S)R}\frac{1}{\lambda+1}. \tag{9}$$

In this equilibrium, consumer welfare is

$$U = \frac{\gamma_S R}{p_S} = \gamma_S R\frac{\lambda+1}{\lambda}.$$

[4] The reason is if so, it must be that $\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}$ is finite. Since $\theta = 0$,this implies $p_L \to 0$. However, consumer FOC implies

$$q\frac{1-\theta}{\theta}\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L} \leq (1-q)\frac{\gamma_S R}{p_S} + (1-q)\frac{\theta}{1-\theta}\frac{\gamma_L X}{p_L}$$
$$\underbrace{q\frac{\gamma_L X}{p_L}\left[1 - \frac{1-q}{q}\frac{\theta}{1-\theta}\right]}_{+\infty} + \underbrace{q\frac{1-\theta}{\theta}\frac{\gamma_S R}{p_S}}_{\to+\infty} \leq (1-q)\frac{\gamma_S R}{p_S}$$

which is impossible. Therefore, it cannot be that $p_L \to 0$ and it must be that $b_F = \gamma_L X$.

Producer profits are

$$\Pi = \frac{(1-\gamma_S)R}{1-p_S} = (\lambda+1)(1-\gamma_S)R.$$

The short asset dominates because, given low long pledgeability, far too much producer capital is required to be allocated to long assets for them to offer producers the same return as short assets. Conversely, the implied shadow price of the long financial claim is too high for consumers to prefer them to the short claim. With limited producer capital relative to consumer capital (λ is large), producers find it more profitable to produce short assets exclusively.

3.2.2 Short glut ($b_F = \gamma_L X$)

As γ_L rises further and $\gamma_L \in [\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, \underline{\gamma}_L]$,[5] some long externally financed assets will be produced.

An increase in γ_L increases the share of long output that can be pledged to households and the fraction of each unit of long investment that can come from households, p_L,while offering the same return as short claims. It commensurately reduces the fraction that must come from producers, $(1-p_L)$. Nevertheless, in the short glut region, there is still a relative shortage of producer capital given how much producer capital each long asset needs, so the producer cannot produce too many longs that are attractive to consumers. Because only relatively small amounts of the long asset are still being produced, consumers mainly hold short assets and the interim price of the long asset will be capped at $b_F = \gamma_L X$, its future payoff. The date 1 to 2 gross interest rate is 1. This implies that for the consumer, the returns on claims held to maturity are equal, i.e., $\frac{\gamma_S R}{p_S} = \frac{\gamma_L X}{p_L}$. So allocation to shorts and longs vary to get the right equilibrium price ratio. In this region, both assets are produced and both claims held, so both producers and consumers must be indifferent and $\frac{1-p_L}{1-p_S} = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}$ and $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$ must be satisfied.

Since $p_L = \frac{1}{1+\frac{y_L}{\lambda\theta}}$ and $p_S = \frac{1}{1+\frac{1-y_L}{\lambda(1-\theta)}}$, the latter (consumer indifference) condition becomes

$$\frac{\gamma_L X}{\gamma_S R} = \frac{1+\dfrac{1-y_L}{\lambda(1-\theta)}}{1+\frac{y_L}{\lambda\theta}}$$

We will refer to this condition as the *capital allocation constraint*, because it essentially is a constraint on the ratio of producer to consumer capital so that both financial claims are attractive to consumers. So when γ_L is low relative to γ_S, as it is when the system is in this

[5] $\underline{\gamma}_L$ solves $X\left(\lambda(1-q)-(\lambda+1)\gamma_S\right)\gamma_L^2 + \gamma_S\left(R(\lambda(q-1)-1)+\lambda q X+(\lambda+1)R\gamma_S+X\right)\gamma_L - qR\lambda\gamma_S^2 = 0.$

region, it must be that $\frac{y_L}{\lambda\theta}$ is high relative to $\dfrac{1-y_L}{\lambda(1-\theta)}$.

Substituting $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$ into the producer's indifference condition and rearranging, we get the prices where both producers and consumers are indifferent:

$$\begin{aligned} p_S &= \frac{\gamma_S}{X}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)} \\ p_L &= \frac{\gamma_L}{R}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)}. \end{aligned}$$

Proposition 2. *If returns and pledgeability are aligned so that $\gamma_S R \leq \gamma_L X$, then short dominance and short glut are impossible.*

Proof. In the case of short glut, we just showed

$$\begin{aligned} p_S &= \frac{\gamma_S}{X}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)} \\ p_L &= \frac{\gamma_L}{R}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)}. \end{aligned}$$

A producer's return must also be strictly above X, the return from retention. Therefore, $\frac{(1-\gamma_L)X}{1-p_L} > X \Rightarrow p_L > \gamma_L$. Therefore

$$p_L = \frac{\gamma_L}{R}\frac{(X(1-\gamma_L) - R(1-\gamma_S))}{(\gamma_S - \gamma_L)} > \gamma_L,$$

which is true only if $\gamma_S > \gamma_L$. This is because producers would strictly prefer long assets if they were equally pledgeable and consumers require the same return on each. Given $\gamma_S > \gamma_L$, it must be that:

$$\begin{aligned} (1-\gamma_S) &< (1-\gamma_L) \\ \Rightarrow (1-\gamma_S)R &< (1-\gamma_L)X. \end{aligned}$$

The non pledgeable return on more profitable long assets exceeds that of short assets because they allow a smaller fraction return to be pledged.

Then, from producer indifference $\frac{1-p_L}{1-p_S} = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}$, we know it must be that

$$1 - p_S < 1 - p_L \Rightarrow p_L < p_S.$$

Then, from consumer indifference $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$, we know it must be that pledgeability and total returns are misaligned so $\gamma_L X < \gamma_S R$. From above, note also that $(1-\gamma_S)R < (1-\gamma_L)X$ and pledgeable and non pledgeable returns are also misaligned. In the short dominance region, γ_L is even lower so $\gamma_L X < \gamma_S R$ and $(1-\gamma_S)R < (1-\gamma_L)X$ must also hold in that case. □

Comparative Statics with respect to γ_L

Lemma 4. *In the short glut equilibrium, as γ_L increases: y_L increases , θ increases, $\frac{y_L}{\theta}$ decreases , $\frac{1-y_L}{1-\theta}$ increases, p_S increases , p_L increases , $\frac{\gamma_L}{p_L}$ decreases . consumer welfare U decreases , producer profits Π increases , and total welfare $\lambda U + \Pi$ increases .*

Proof. See Appendix. □

Discussion:

As γ_L rises, more of the return from long assets can be paid out to financial claims. Although this reduces the producer's rent from financing associated with long claims, the producer claim per unit of capital invested in long claims still exceeds that on short claims: $(1-\gamma_S)R < (1-\gamma_L)X$. Consequently, producers have an incentive to shift towards producing long claims. Since the capital-constrained producer can produce less than one unit of long asset for every unit reduction of short asset (since $1-p_L > 1-p_S$), and because long assets are less pledgeable (that is, $\gamma_L X < \gamma_S R$),overall future amounts that can be pledged to consumers falls. Given fixed consumer capital up front, and equal returns across financial claims, it must be that consumer returns fall and consumers are worse off as they too shift capital to longs. By contrast, producers benefit from this change because they produce more long assets and receive higher prices for their issued financial claims, increasing their profitability. From an aggregate perspective, since more long assets are produced from the available resources, total welfare increases.

3.2.3 Illiquid Long

With an increase in γ_L in the short glut region, more units of long assets are produced relative to short assets. Eventually, sufficient long financial claims are produced relative to short so the purchasing power at date 1 of late consumers (the buyers) is less than the future value of late claims sold by early consumers. As a result, $b_F = \min\left\{\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}, \frac{\gamma_L X}{p_L}\right\} < \frac{\gamma_L X}{p_L}$. Now the date-1 price on the long is less than face value, which means longs are illiquid. In addition, the capital allocation constraint that consumer returns on short financial claims is as large as that on long (which was binding when we were in the short glut region), is no longer binding. Long financial claims now offer strictly more held to maturity than short financial claims. In this case, the consumer's asset allocations are set anticipating the returns from their trades of long and short assets at date 1, which implies $\theta = q$, as we have explained in subsection 2.5, so consumer allocations to each asset are fixed in this region. Given so, $p_L = \frac{q\lambda}{(q\lambda + y_L)}$ and $p_S = \frac{\lambda(1-q)}{\lambda(1-q)+(1-y_L)}$, prices are fully determined by producer allocations. Further substituting these prices into the producer's FOC (1), we get a quadratic in y_L, $\frac{(1-\gamma_L)X}{\lambda(1-q)+(1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\lambda q + y_L} y_L$.

Comparative Statics with respect to γ_L

Lemma 5. *In the illiquid long with rent equilibrium, as γ_L* increases*: y_L decreases, p_S decreases, p_L increases, and $\frac{\gamma_L}{p_L}$ increases . Consumer welfare U increases, producer profits Π decreases , and total welfare $\lambda U + \Pi$ decreases with γ_L.*

Proof. See Appendix. □

Discussion:

As long pledgeability improves, thus increasing the date-2 payment to consumers, allocations to the long asset, producer welfare and equally weighted overall welfare fall. The key difference here from the short glut region is that consumer allocations to claims do not change with γ_L, otherwise the ability of late consumers to trade short for long at date 1 would result in one claim dominating the other – it is only when $\theta = q$ that the ex-ante returns on the claims are equalized. Producer allocations are therefore dispositive here. So when γ_L goes up, non-pledgeable long producer returns fall and producer investment in the long asset, y_L, must go down. Intuitively, to restore producer incentives to invest in the long asset, it must be that $p_L(= \frac{\lambda q}{\lambda q + y_L})$ increases, which can only be if the producer invests less in the long asset, that is, y_L falls.

Consequently, $p_S = \frac{\lambda(1-q)}{\lambda(1-q)+(1-y_L)}$ falls (since the producer invests more in the short), so that consumer returns from shorts, $\frac{\gamma_S R}{p_S}$, increases with γ_L. So in the new equilibrium, the producer's return from producing shorts, $\frac{(1-\gamma_S)R}{1-p_S}$, falls, so too must the producer's return from producing longs, $\frac{(1-\gamma_L)X}{1-p_L}$(despite the increase in p_L). This must imply that the consumer's return from holding long $\frac{\gamma_L X}{p_L}$ increases (because p_L increases by less than γ_L). Note that different from the short glut region, consumer returns from both claims increase – the long claim because it becomes more pledgeable so larger payoffs offering higher returns are available for sale, reducing producer rents from financing and increasing consumer returns, and the short claim because the producer shifts to producing more of it, reducing prices per pledgeable payoff (given the consumer does not shift allocations). Overall output and welfare is fully determined by producer allocations, and welfare falls. Since consumer returns increase on both claims and the consumer's allocations do not change, consumer welfare increases. Importantly, as we will see, an increase in pledgeability of any asset in this region tends to reduce producer returns, and pushes the producer to produce more of the asset whose pledgeability has not increased in order to limit the fall in producer returns. This seemingly counter-intuitive effect of higher pledgeability on an asset's production are primarily because the possibility of interim trade forces consumer allocations stay constant to prevent arbitrage. Consequently, since consumer allocations do not shift towards the more pledgeable asset to enhance its price, higher pledgeability for an asset directly reduces the producer's return from producing the asset.

3.2.4 No Long Rent ($p_L = \gamma_L$)

As γ_L rises further in the illiquid long with rent region, p_L rises but at a slower rate and eventually meets γ_L from above. At this point, the rent from financing the long asset falls to zero because the price at which the long claim is sold to consumers is exactly equal to its long pledgeability – so all returns are passed through to the consumer. The return to consumers from investing in the long claim tops out at X, the same return as when the producer invests in the long asset entirely with own funds (retention), or with external financing:

$$p_L = \gamma_L \Rightarrow \frac{(1-\gamma_L)\,X}{1-p_L} = X.$$

Since the producer's return on the long asset is X, the producer's FOC requires this to be the return on producing the short asset whenever $\gamma_S < 1$, which implies

$$p_S = 1 - (1-\gamma_S)\,\frac{R}{X}.$$

It is easily checked that the return to the consumer is below R from investing in the short financial claim, while the return on the long financial claim is X. Yet the consumer's expected utility from either claim is equal because the long claim is illiquid. In this region, only changes in short pledgeability can change the rate of return available to consumers. Note that $y_S + y_L \leq 1$ and $1 - y_S - y_L$ is used to produce and retain long. The consumer again invests $\theta = q$ to avoid arbitrage profits from trade at date 1.

Comparative Statics with respect to γ_L

Lemma 6. *In the no long rent region, y_L decreases with γ_L, y_S is unchanged with γ_L so producer retention goes up with γ_L. θ and p_S are independent of γ_L, p_L increases with γ_L, and $\frac{\gamma_L}{p_L}$ is unchanged with γ_L. Consumer welfare U, producer profits Π, and total welfare $\lambda U + \Pi$ are all unchanged with γ_L.*

Proof. See Appendix □

Discussion:

In the no long rent region, the producer makes no rents on selling claims on the long asset, so she is in indifferent between self financing it or partially financing it outside. The limited pledgeability of the long asset does not constrain the pricing or production of long financial claims. Furthermore, the rate of return on producer capital invested in the short asset is also fixed to equal that of producing the long asset, X. This is, the producer earns no rent on producing short claims and short claims have consumer returns below R only because of the opportunity cost of investing in an asset with return below longs, because R<X. Since an increase in the pledgeability of the long asset only reduces producer allocation to externally financed production but not overall production of the long asset, it has no effect on producer welfare. The consumer's allocations are also fixed, and her return on the long claim is fixed. So overall welfare does not change with changes in long pledgeability.

3.2.5 Conditions for all cases

We derive conditions for the various regions to exist if $\gamma_L$is allowed to vary.

1. $X < (\lambda(1-q)+1)(1-\gamma_S)R$. There is not a *no long rent* region.
   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}]$: short dominance
   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, \underline{\gamma}_L]$: short glut
   (c) $\gamma_L \in [\underline{\gamma}_L, 1]$: illiquid long.
2. $X > (\lambda+1)(1-\gamma_S)R$. There is not a *short dominance region*
   (a) $\gamma_L \in [0, \underline{\gamma}_L]$: short glut
   (b) $\gamma_L \in [\underline{\gamma}_L, \frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}]$: illiquid long t
   (c) $\gamma_L \in [\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}, 1]$: no long rent
3. $(\lambda(1-q)+1)(1-\gamma_S)R < X \leq (\lambda+1)(1-\gamma_S)R$. All four regions exist
   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}]$: short dominance
   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, \underline{\gamma}_L]$: short glut
   (c) $\gamma_L \in [\underline{\gamma}_L, \frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}]$: illiquid long
   (d) $\gamma_L \in [\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}, 1]$: no long rent

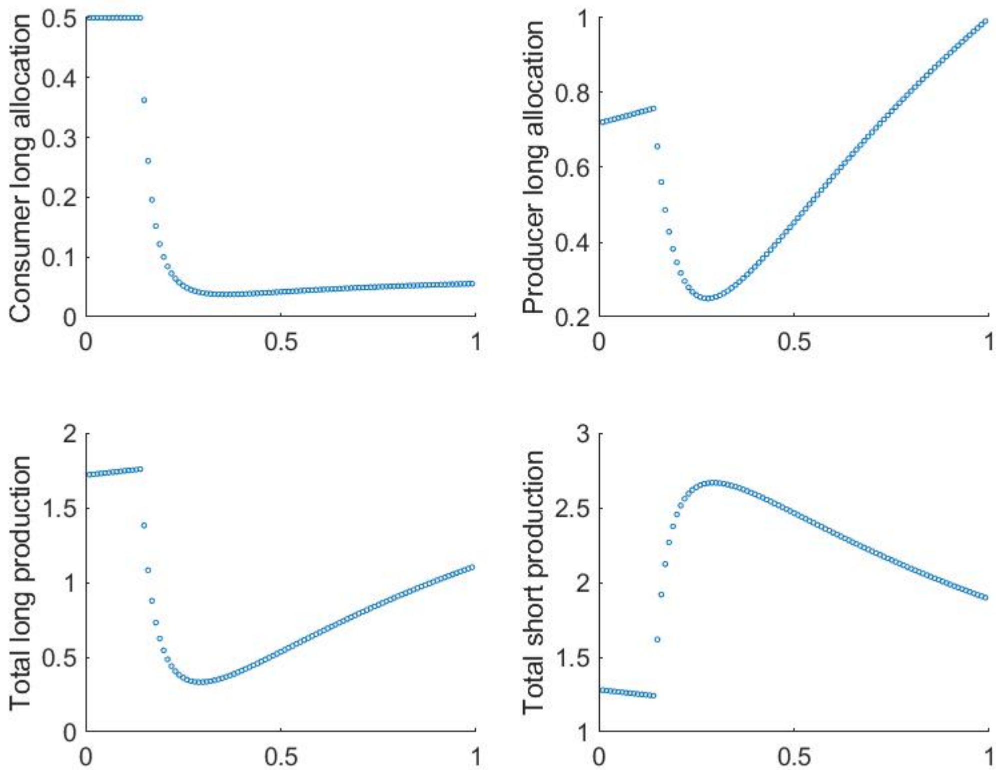
3.3 Credit development

Having seen how changes in γ_L affect outcomes, we now analyze how an increase in short pledgeablity γ_S affects the equilibrium outcome. An increase in short pledgeability, γ_S, could be thought of as improved working capital lending, for instance stemming from the greater transparency and enforceability of short term commercial paper or bills of exchange, or bank verification of cash receipts. Other examples include easier borrowing against inventory, facilitating trade and commerce. All these are aspects of *credit development*.

Our previous analysis offers another way to see the intuition behind our results. An increase in short pledgeability will increase the *financeability* of the short asset relative to the long asset. Ordinarily (though not always), this should increase consumer allocations to the short asset issued, increasing the producer's incentive to produce more of it. At the same time, an increase in short pledgeability will reduce a producer's financing *rents* associated with the short asset relative to the long asset. Ordinarily (though not always), this should reduce the producer's incentive to produce more of it. Outcomes depend on how financeability trades off against rents.

We will see that increased short pledgeability always makes the consumer better off, and makes the producer (weakly) worse off. The effects on overall welfare are, once again, more ambiguous. An example may be useful to set ideas.

Figure 2: Allocation and Production under different γ_S



In this example (see Figure 2), the decentralized equilibrium is in the *illiquid long* region with γ_S below 0.14. An increase in short pledgeability does reduce the producer return from producing short assets, ceteris paribus. Since consumers do not reallocate in this region (consumer's allocation stays unchanged at $\theta = q$), the producer shifts allocations toward the long asset. As γ_S rises above 0.14, the equilibrium shifts to *short glut*. Producer allocations to long assets fall initially with γ_S, but have a non-monotonic pattern, reaching bottom at$\gamma_S = 0.28$, while consumer allocations to long reach their minimum at $\gamma_S = 0.35$. The total-produced long asset also first declines, reaching its minimum at approximately $\gamma_S = 0.29$, before moving up. We will explain these patterns shortly.

Figure 3: Welfare under different γ_S

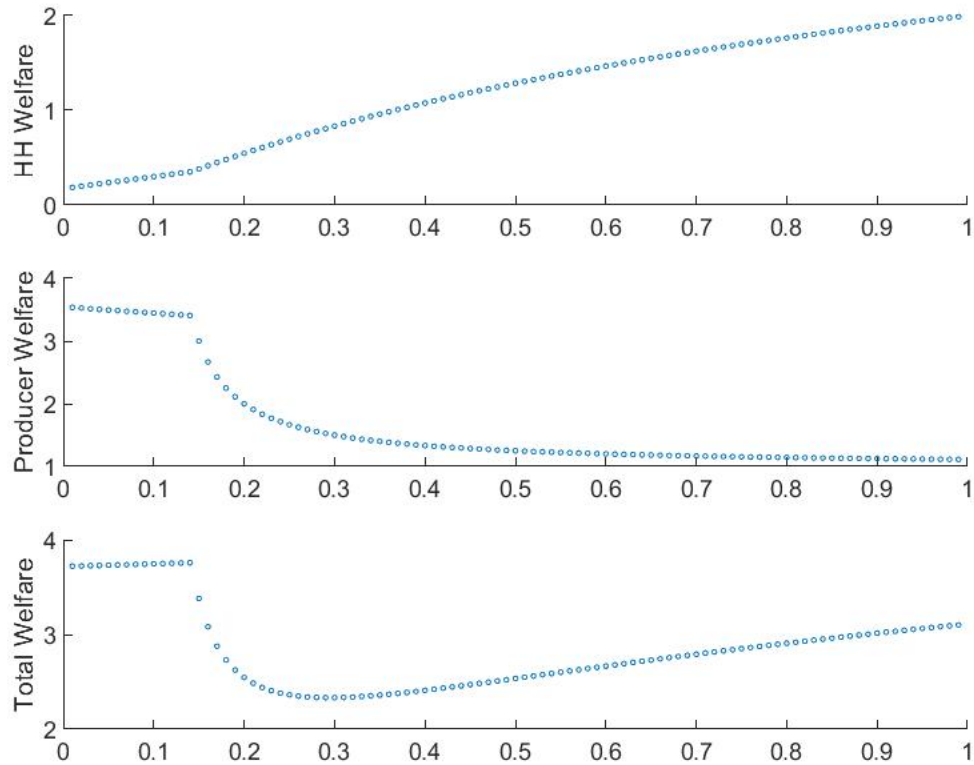


Figure 3 plots the payoff of different parties as γ_S increases. Since consumer returns increase throughout, consumer welfare increases monotonically. Total welfare is non-monotonic in the short glut region, given that total long production is also non-monotonic. It turns out that producer payoffs are monotonically decreasing. Now let us understand the outcomes in the possible regions in the more general case (see Figure 1).

3.3.1 Illiquid long

If short pledgeability is sufficiently low or producer capital is sufficiently high, the economy will be in the illiquid long region. In this case, the possibility of trading to buy the illiquid long at date 1 implies that consumer allocations are fixed at $\theta = q$. An increase in γ_S will reduce the producer rent from financing shorts, without any ability to attract more capital to shorts, producers will substitute toward producing longs. Since the shift in producer allocations to the long asset reduces p_L (given fixed consumer allocations), consumer returns increase on the long claim. This must reduce producer returns on the long asset, and since producer returns on both assets are equal in equilibrium, producer returns on the short asset must fall in equilibrium. So consumer returns on the short claim must rise. Consequently, consumer welfare increases, while producer welfare falls. As a result of the increase in producer allocations to long production, overall welfare increases. So an increase in the pledgeability of the low-return short asset improves allocations and overall welfare, but not producer profits. Recall that improving long pledgeability in this region also reduced producer profits while increasing consumer welfare, but reduced overall welfare because allocations to the long asset fell.

Lemma 7. *In the illiquid long asset region, y_L increases with γ_S, p_S increases with γ_S, and p_L decreases with γ_S. Consumer welfare U increases with γ_S, producer profits Π decreases with γ_S, and total welfare $\lambda U + \Pi$ increases with γ_S.*

Proof. See Appendix. □

At higher levels of γ_S, *ceteris paribus*, pledgeable returns will become misaligned. so that $\gamma_S R > \gamma_L X$ and $X(1-\gamma_L) > R(1-\gamma_S)$. The short dominance and short glut regions become possible. If $\gamma_L$is low, increases in γ_S take the regime into the short glut region. For much higher levels of γ_L, the system moves instead into the no rent region.

3.3.2 Short glut region

Suppose that one is in the short glut region. For consumers to hold both claims after an increase in γ_S, $\frac{p_L}{p_S}(=\frac{\gamma_L X}{\gamma_S R})$ should fall. Think of this as relative financeability. At the same time, from the producer's perspective, $\frac{1-p_L}{1-p_S}(=\frac{(1-\gamma_L)X}{(1-\gamma_S)R})$ should increase. Think of this as relative producer rents. Both conditions can be met with a fall in p_L and a rise in p_S as γ_S rises.

If $\gamma_S$is low for any level of γ_L which is not too low (it must be somewhat low relative to γ_S for the economy to be in the region), an increase in γ_S will have more effect on relative financeability and little effect on relative producer rents. It makes sense for the producer to shift to producing more short assets, with consumers allocating more capital to short claims, away from long claims. Given that each unit of long releases more producer capital than each unit of short (recall $1-p_L > 1-p_S$ in this region), and vice versa for consumer capital, it must be that a disproportionate amount of consumer capital leaves longs, pushing down p_L. So returns to consumers from holding longs will increase in the new equilibrium. Of course, for producers to see a financing reason to shift allocations, it must be that p_S rises, but in equilibrium, the consumer returns to holding shorts must rise to equal the returns to holding longs, so $\frac{\gamma_S}{p_S}$ increases with γ_S.

As γ_S rises further, an increase in γ_S reduces relative producer rents significantly while not increasing relative financeability as much. The trade-off shifts. This is when the producer starts increasing long production with further increases in γ_S. So while each unit of short not produced allows less than one unit of long to be produced because of producer capital constraints, the released consumer capital has to pay both for the more pledgeable short claims and the additional long claims. Given the limited consumer capital, consumer returns continue rising. Eventually, more consumer capital could start being allocated to longs as short production falls off sufficiently. In all situations in the short glut region, consumer returns always increase with γ_s due to increasing the pledgeable return on the more pledgeable asset.

Lemma 8. *In the short glut equilibrium, p_L decreases with γ_S, and $\frac{\gamma_S}{p_S}$ increases with γ_S, consumer welfare U increases with γ_S, producer profits Π decrease with γ_S. Total welfare $\lambda U + \Pi$ is non-monotonic in γ_S.*

Proof. See Appendix. □

From the short glut region, as Figure 1 suggests, at low levels of γ_S an increase in γ_S can cause a transition to the short dominance region. This is because the increase in financeability dominates the increase in relative rents, and short production dominates.

3.3.3 Short dominance

In the short dominance region, an increase in γ_S reduces producer rents from creating shorts, without altering production because the producer returns to producing shorts dominate the returns to producing longs within the region. Consumers are better off (their return increases to $\frac{\lambda+1}{\lambda}\gamma_S R$) while producers are worse off, and overall welfare is unchanged.

Eventually, as γ_S rises to high levels, the relative rent effect dominates and we leave dominance and enter the short glut region. Producers earn so little from long assets that they return to investing in long assets in a small quantity to allow the long claims to offer consumers the same return as short.

3.3.4 No rent

Finally, in the no rent region (which occurs when both $\gamma_L$and γ_s are high), the producer makes no rents selling claims on the long asset, so she is in indifferent between self financing it or partially financing it outside. Furthermore, the return on producer capital on the short asset is also fixed to equal that of producing the long asset, X. So changes in short pledgeability affect producer allocations, but not producer welfare. An increase in short pledgeability allows the producer to allocate more to the self-funded long asset. So her allocation to short production falls. The consumer's allocations are fixed at $\theta = q$, and his return on the long claim is fixed. With the increase in short pledgeability, the price of the short claim rises but by less than the increase in γ_S, so consumer returns rise. As a result, the consumer is better off – essentially her gains come from the greater overall allocation to the higher return long asset from the more pledgeable short asset. Importantly, the illiquid long no rent region is where neither producer nor consumer wants to stand in the way of increased pledgeability of either asset, and total welfare will also increase with short pledgeability. Put differently, conflicts of interest fall off at high levels of long pledgeability, in part because changes in pledgeability do not cause perverse changes in financing rents and hence allocations.

Lemma 9. *In the no rent region, y_L is unchanged with γ_S, and y_S decreases with γ_S so producer self-financed long goes up with γ_S. θ and p_L are independent of γ_S. p_S increases with γ_S, and $\frac{\gamma_S}{p_S}$ increases with γ_S. Consumer welfare increases with γ_S, producer profits Π are independent of γ_S. Total welfare $\lambda U + \Pi$ increases with γ_S.*

3.3.5 Discussion

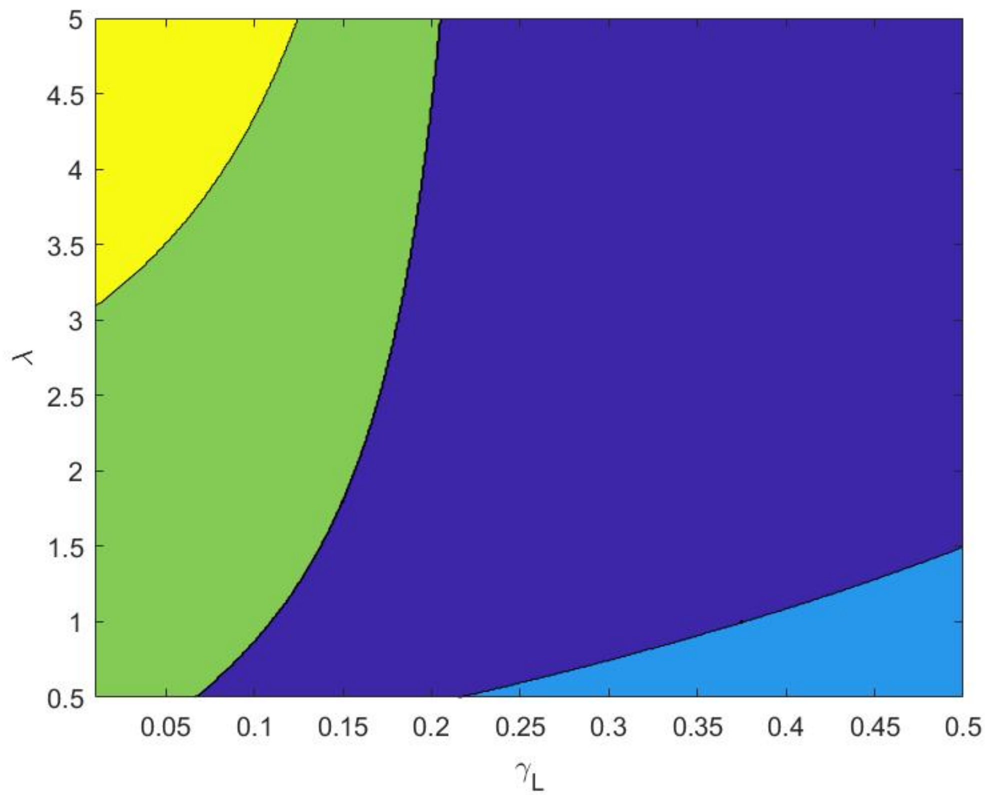
As we see from the various cases, the effects of financial development differ depending on whether it improves short-term or long-term pledgeability. An improvement in short pledgeability, that is, credit development, tends to increase consumer welfare, but decrease producer welfare. Outside the short glut region, overall welfare increases. Intuitively, an increase in short pledgeability allows the producer to save on scarce capital allocated to the lower return short asset, and instead use it to produce the higher return long asset. In the short glut region, however, it is not always the case that an increase in short pledgeability increases the producer allocation to the long asset, which is why welfare may also fall.

Contrast this with long pledgeability, whose increases tends to reduce the attractiveness to the producer of producing the long, welfare-enhancing, asset, while also reducing the quantum of producer capital needed to produce each unit of it. The exception to the rule that higher pledgeability hurts the producer once again is in the short glut region, where an increase in long pledgeability from low levels allows larger consumer allocations to long claims, and increases the attractiveness to the producer of producing long assets by enhancing financing rents and reducing the producer capital needed per unit of long

3.4 The increase of consumer capital relative to producer capital

Let us turn finally to changes in the relative amount of consumer capital relative to producers, λ. Figure (4) plots the equilibrium region as γ_L and λ vary. The yellow region is dominant short asset, green is short glut, dark blue illiquid long with rent and light blue illiquid long no rent. We prove in the appendix that the thresholds in γ_L for different regions all increase with λ.

Figure 4: Equilibrium Cases as a function of γ_L and λ



The result under $\lambda \to 0$ is straightforward: specifically, the equilibrium must be one with no long rent. In this case, $p_L = \gamma_L$, $p_S = 1-(1-\gamma_S)\frac{R}{X}$, $y_L \to 0$, $y_S \to 0$, and $b_F = \frac{\gamma_L \gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}$. When λ is small, and thus producer capital relatively plentiful, at the margin producers will self finance the production of the long asset, since external financing is small and not a source of rents. Essentially, producers have more capital than required to fund long investment, given what consumers will pay for the pledgeable portion. Competition will have producers pass through the long rate of return, X. However, at higher values of λ, returns to consumers will fall (because there is insufficient producer capital to eliminate rents). This will introduce trade-offs for producers which we highlighted in the previous sections.

In the short dominance region, an increase in λ will not change allocations of course (all capital is invested in shorts), but will increase p_S since more short financial claims have to be sold without any additional producer capital – the price of short financial claims will have to rise to ensure additional investment can be fully financed. Consumer returns fall. Turn next to the short glut region.

Lemma 10. *In the short glut region, y_L decreases with λ, θ decreases with λ, p_S and p_L are independent of λ. Consumer welfare U and producer profits Π are independent of λ.λ.*

Proof. Clearly, the closed-form solutions for the fractions of consumer capital backing each asset, p_S and p_L, derived in section 3.2.2 show that both are independent of λ. From $\theta = \frac{\frac{1}{\lambda} - \frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L} - \frac{1-p_S}{p_S}}$, we know that θ decreases with λ. From $y_S = \frac{\lambda(1-\theta)(1-p_S)}{p_S}$, we know y_S must increase with λ, so that $y_L = 1 - y_s$ decreases with λ. Consumer welfare $U = \frac{\gamma_S R}{p_S}$, producer profits $\Pi = \frac{1-\gamma_L}{1-p_L} X$ are both independent of λ.

The financing of each unit of production, and the price of that financing does not change. Increases in λ imply that producer capital falls relative to consumer capital, so the producer has to move towards producing the asset that enables greater external financing per unit to accommodate the greater availability of consumer capital. In this region, this is the short asset.

Turn next to the illiquid long with rent region. From the producer's FOC,($(1 - p_L) = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1 - p_S)$. So the producer deploys less capital per unit of longs iff $(1 - \gamma_S)R > (1-\gamma_L)X$. In this case, an increase in λ will mean more producer capital will be allocated to the asset that requires less producer capital per unit so that the additional consumer capital can be absorbed without any change in the relative fraction of producer capital. □

In the short glut region, both producers and consumers must be indifferent and $\frac{1-p_L}{1-p_S} = \frac{(1-\gamma_L)X}{(1-\gamma_S)R}$ and $\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$ must be satisfied. Therefore, the allocations p_L and p_S are independent of λ. Market clearing requires

$$\lambda \left(\frac{1-p_L}{p_L}\theta + \frac{1-p_S}{p_S}(1-\theta) \right) = 1,$$

so that θ decreases with λ. Finally, from $p_L = \frac{1}{1+\frac{y_L}{\lambda\theta}}$, we know that y_L must decrease with λ as well.

Lemma 11. *In the illiquid long region, y_L increases with λ if and only if $(1 - \gamma_S)R > (1 - \gamma_L)X$. θ is independent of λ.*

Proof. See appendix. □

Finally, in the no rent region, to take advantage of the greater external financing made possible by the availability of consumer capital, the producer will reduce self-financing and instead finance more production externally. As she increases the number of externally financed long units, she also will have to increase the number of externally financed short units so as to keep the interim price, b_F, constant. Hence we have

Lemma 12. *In the illiquid long no rent equilibrium, both y_L and y_S increases with λ. θ, p_L, and p_S are independent of λ.*

Below, we fix γ_L and vary λ. As λ increases, the equilibrium travels across illiquid long, short glut and finally dominance. Figure (5) describes the allocation, whereas (6) shows how welfare changes.

Figure 5: Allocation and Production under different λ

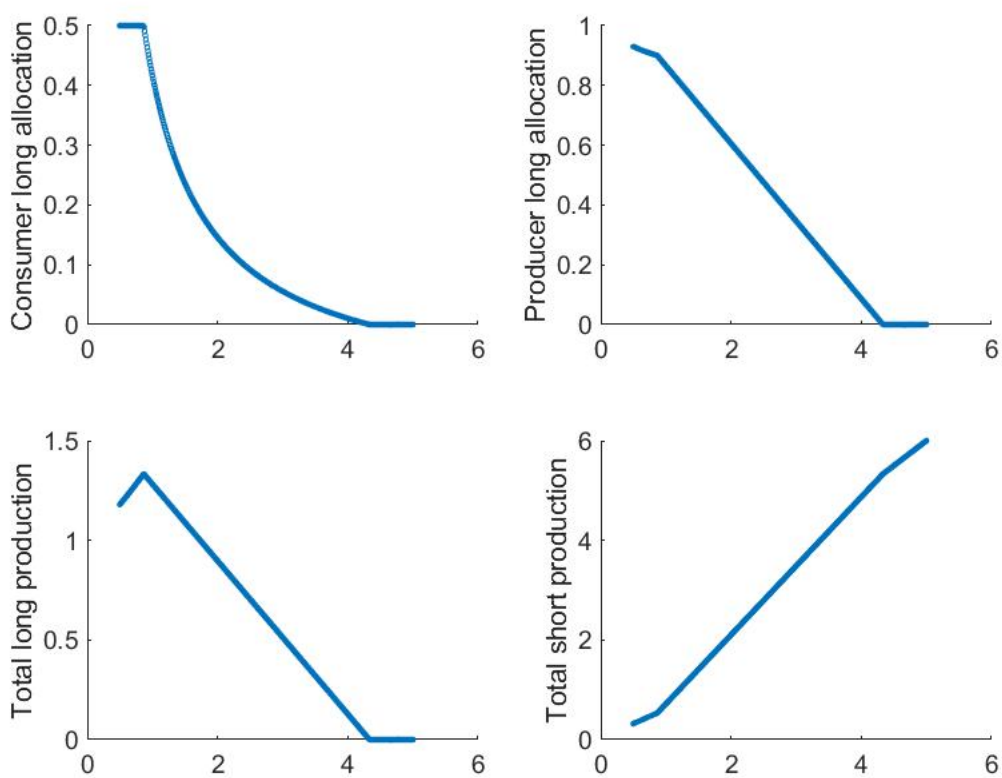
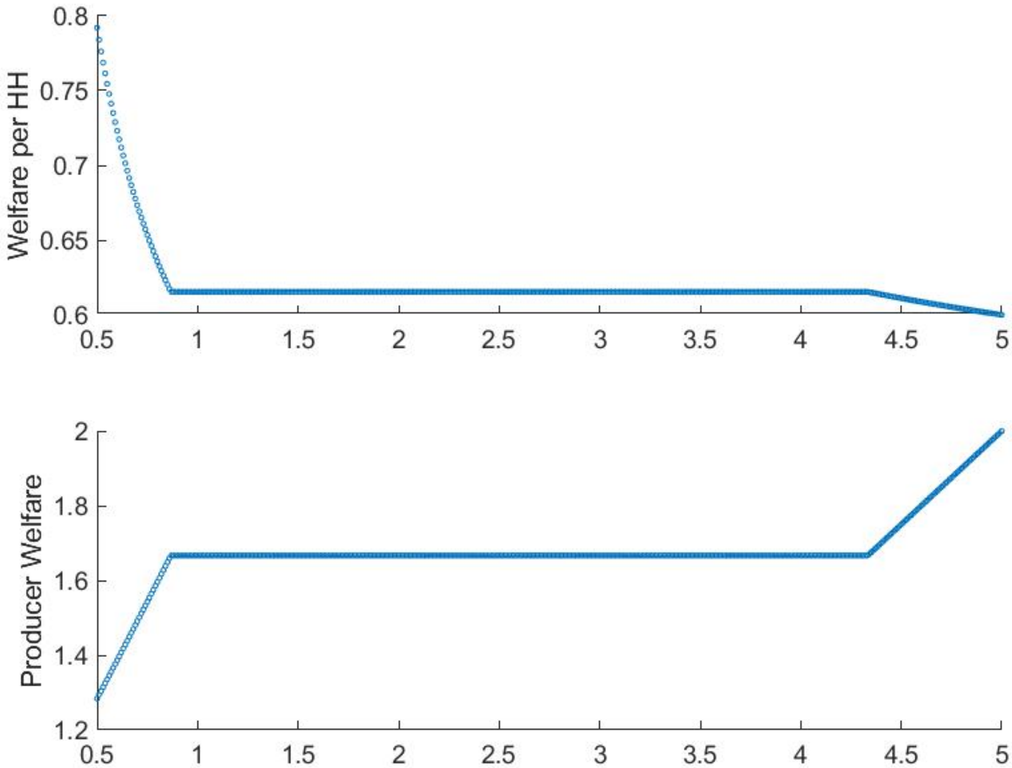


Figure 6: Welfare under different λ



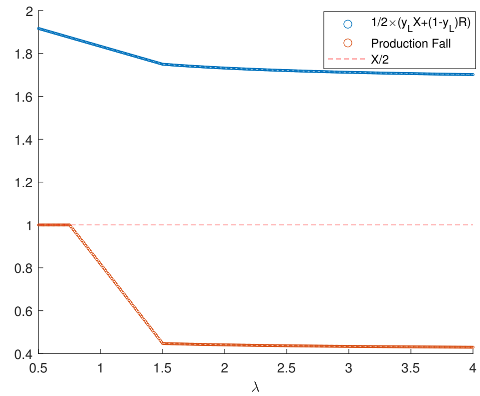
Discussion:

In economies where producer capital is rather high and with where either the rents from producing longs exceed shorts $(1-\gamma_L)X > (1-\gamma_S)R$ or there where there are no rents, a reduction in producer capital can result in a movement to short assets and claims. An abrupt increase in λ while γ_L and γ_S are unchanged could be thought of as an economic crisis or the advent of new sources of competition (for instance, imports), reducing the capital of producers relatively more than that of consumers. The net effect is to move allocations away from long assets, with attendant effects on output and productivity (as in Eisfeldt and Rampini).

We consider a case where producer capital becomes $\frac{1}{2}$. The rest of the parameters are as follows: $X = 2$, $R = 1$, $q = 0.5$, $\lambda = 2$, $\gamma_L = \gamma_S = 0.5$. In this economy, the equilibrium is Illiquid Long Asset with Financing Rent. Total long production is 1.3508, and short is 1.1492. If total producer is 1, the equilibrium is still Illiquid Long Asset with Financing Rent. Total long production is 1.7321, and short is 1.2679. The difference in overall production is

$(1.7321 + 1.2679 - 1.3508 - 1.1492) = 0.5$, which is exactly $\frac{1}{2}X$. Next, we plot the difference as λ varies.

Figure 7: Welfare under different λ



4 Implications for financial and credit development

Institutional development is generally thought to be good – more of it gives society more tools, more contractabiliy, more ability to commit – and all this enhances the pace of economic growth and well-being. We introduce two elements that are typically overlooked in models of development – that production may be driven by a specialized group that enjoys rents from offering financial claims to the consuming population, and that different maturities of production have different returns and different degrees of pledgeability. In this environment, institutional development need not make everyone better off, and need not even make society better off. What is particularly interesting is who benefits and who loses from development, and under what circumstances. This then can sharpen the predictions of our model and allow us to suggest the kinds of systems of government that can be most friendly to development at different stages of development.

4.1 Technologies: short term vs long term

The two technologies are the ones that typically confront developing societies – produce tradeable goods over the short term with easy-to-understand, transparent technologies, and with output generated in the short term making it easier for producers to commit to pay outside financiers, or produce goods over the long term with more complicated technologies, with the longer duration between inputs and final production making it harder for producers to commit to pay outside financiers. There are other ways of describing these technologies –

for instance, primary sector goods produced for short term trading (grain, fruit and vegetables, gold and silver) vs sophisticated secondary or tertiary sector goods produced as a result of long term investment (cycles, cars, and reliable pension funds). Short-term pledgeability increases, for instance, when enforcement of short term repayment including the monitoring and seamless seizure of pledged collateral is easier. Long-term pledgeability is improved by better bankruptcy laws, more reliable auditing of company books, a more vibrant market for corporate control, etc.

4.1.1 Short dominance region: Primitive economy and the possibility of development traps

Given these attributes, it seems natural that underdeveloped or primitive societies start off with higher short pledgeability γ_S than long pledgeability γ_L, which itself would be quite low. In such cases, pledgeable returns are likley to be misaligned with overall returns. It is likely that the rate of return on producer capital from short production dominates the return on long production and the economy is in the short dominance region, where only the short asset is produced. The economy is likely to be producing low-return primary sector goods, since the appropriable returns from long term investment for the consumer, and hence the relative returns for the producer from producing it, are perceived to be relatively low.

As we have seen, low producer capital relative to consumers tends to accentuate the impact of cum-financing producer returns in determining capital allocations. So this region is more likely when the producer has little capital relative to consumers. Because the returns from short production are not high, producer capital is unlikely to improve even if we consider a more dynamic setting. Furthermore, with no long term production, there will be little effort to improve corporate governance and other aspects of long pledgeability. On the other hand, consumers do benefit if short pledgeability is enhanced, for it shrinks producer returns from producing the short asset, and enhances consumer returns. Whether short pledgeability is enhanced in this region therefore depends on who is in charge. If producers are (as in a oligarchy), we get a *development trap* where no financial development takes place, if consumers are (as in a democracy), we get *skewed financial development* focused on enhancing credit development.

Historians (see, for example, Braudel (1980)) have emphasized that in the early stages of Western capitalism, entrepreneurs focused on trading output from short production (agricultural commodities or artisanal production from the putting-out system) rather than investing in capital-intensive factories driven by, say, water power. That entrepreneurs in under-developed economies are attracted to lower-return commerce rather than higher return but complex manufacturing is consistent with an environment of low producer capital and relatively low long term pledgeability.[6] Our model suggests the shift from commerce toward manufacturing required (1) producers to become relatively richer (for instance, as a result of the steady accumulation of business profits or as a result of windfalls that benefited the adventurous producer class) (2) the relative pledgeability of long versus short assets to increase, say as a result of institutional development.

4.1.2 Short glut region: Developing country and oligarchic development

If a developing economy starts out with somewhat better prospects – the potential return from long investment is significantly higher than from short investment, but long pledgeability is low and short pledgeability is moderate – returns are still misaligned but the economy is in the short glut region. Both forms of production take place. An increase in long pledgeability increases overall welfare because it increases long production – interestingly in this case because the increased fraction of consumer capital devoted to long financing enhances rents from financing on both long and short production, on long because there is more demand from consumers for long claims, and on short because fewer short assets (and hence claims) are produced. Producers and consumers have diametrically opposed views, therefore, on the desirability of higher long pledgeability. Producers like it because they can sell more financial claims at higher prices, consumers dislike it because it lowers their returns. The converse is the case with increases in short pledgeability, where the producer is worse off, while the consumer is better off.

So who has control matters once again in this region. Where producers have control – for instance, oligarchies – the scarcity of long term production is likely to incentivize the oligarchic government to enhance long term pledgeability, thereby increasing long term production and producer rents, at the expense of the consumer. Conversely, a consumer-oriented government such as a democracy is likely to enhance short term pledgeability, which may have the effect of reducing overall output (if producers switch to short production), even as it benefits consumers. Of course, substantial changes in γ_L or γ_S can move the system out of this regime (see Figure 1), when the incentives to alter pledgeability will change.

4.1.3 Illiquid long region with producer rents: The Middle Income Trap

When long pledgeability increases to the point that the economy moves into the *illiquid long with rent* region, producers no longer want higher long pledgeability. They also do

[6] Of course, institutions can also be weak on the real side. Long, high return production may suffer from a lack of property rights enforcement – complex fixed assets may need more security – which may reduce their returns relative to short duration production.

not want more short pledgeability. In a sense, consumer allocations are fixed by the arbitrage possibilities associated with trade, which reduces any financing benefits of enhanced pledgeability to the producer, while still weighing on financing rents. If producers were in charge, further financial development would stop – a kind of middle income trap. Consumers would want greater pledgeability, and they would implement it if in charge – for instance, in a democracy – but overall welfare would fall if producers shift away from long production. Middle income traps are usually associated with producer rents (see xx), but our model would also point to financing rents helping bolster producer reluctance to undertake reforms. A transition from oligarchy to democracy in this region would enhance financial development and benefit the average consuming citizen at the expense of producers. The effects on overall output depends on which pledgeability is being enhanced (negative in the case of long pledgeability, positive in the case of short pledgeability).

4.1.4 Illiquid long no rent region: The absence of conflicts

When long and short pledgeability get so high that the economy enters the *no long rent* region, further increases in long pledgeability do not affect consumer, producer, or overall welfare, while increases in short pledgeability improve consumer and total welfare. The conflicts of interest over financial development abate (no one is opposed to higher pledgeability). The reason, quite simply, is the distortionary financing rents, and the consequent effects of increases in pledgeability on allocations and rent sharing, are eliminated in the illiquid long no rent region. Impediments to financial or credit development stemming from conflicts of interest fall away.

4.1.5 In Sum

Of course, if producer capital is large relative to consumer capital (that is, λ, is small), producers will put sufficient capital into producing each asset that rents from financing are driven down, and their choice between assets to produce are driven largely by their intrinsic returns and consumer preferences, even if financial development is modest.

All agents are then also more welcoming of higher pledgeability. Consequently, our analysis suggests that financial development is easier for countries at a higher level of development, in part because producers are wealthier and their decisions are affected less by financing rents, and in part because beyond a certain threshold, financial development itself drives down financing rents and conflicts of interest over them (that is, the system moves into the no rent equilibrium where conflicts of interest over development are low). But getting there from another equilibrium is difficult, which is why financial development is unlikely to be a linear process.

5 Related Literature(a section in progress)

There is a large literature on limited pledgeability and the role of the net worth of producers in facilitating investment. Import studies include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Hart and Moore (1994) and Holmström and Tirole (1998). A bit closer to our model is the literature on financial intermediary capital, where some assets are best held by financial intermediaries and their net worth determines if they are able to hold the asset which helps determine the asset's price. Key work in this area include He and Krishnamurthy (2013); Holmstrom and Tirole (1997); Rampini and Viswanathan (2019). These models focus on how low intermediary capital prevents an institution from providing its important service (monitoring or superior collateralization), rather than effects on the relative impact of low intermediary capital on the relative profitability of multiple assets which it could provide the important service.

There are previous studies that examine investment in assets which vary in their pledgeability but have identical maturity. Our model has similarities to Matsuyama (2007), who examines an economy where indivisible projects differ in both pledgeability and overall productivity. Producer capital really matters when higher productivity projects have lower pledgeability, since they need more own-financing to be undertaken. When producer capital is low, more pledgeable but low return projects are undertaken because they require less producer capital, but this ensures producer capital does not grow, suggestive of a credit trap. Conversely, a producer with more capital can undertake more productive projects, funding the shortfall given their low pledgeability with own capital, generating higher future capital. Higher producer capital therefore implies higher productivity and growth. In Matsuyama (2007), the most attractive project, taking into account both productivity and pledgeability, attracts all the funding. So undoubtedly, an improvement in the pledgeability of the most productive project must improve its chances of being undertaken, and hence overall productivity. However, an improvement in the pledgeability of less productive projects can also improve their chances of being undertaken, in this case reducing productivity. So financial development is not always good.

Unlike Matsuyama (2007), we allow for both types of projects to be undertaken simultaneously, and for project maturity to also matter. We show that high productivity long term projects with higher-than-short pledgeability may still coexist with short projects, with the latter valued for liquidity. Unlike Matsuyama (2007), we also show that an increase in the pledgeability of the high productivity long project can reduce welfare because producers produce less of it given their diminished rents from financing. Conversely, an increase in the pledgeability of the lower productivity short project can improve welfare because the economy can generate the needed liquidity with fewer low productivity projects. The difference in our results derive, of course, from differences in our models.

In a dynamic model which shares features with ours, Ebrahimy (2022) examines the choice of producer investment when producers have the choice between high return low pledgeability projects and low return high pledgeability projects. Unlike us, he does not allow investors to differ in their consumption preferences, or for projects to differ by maturity, and hence for investors to have a choice between claims of different maturity. Ebrahimy (2022) shows that an increase in the pledgeability of the low return project, a form of financial development, can move the economy away from the social optimum, as more is invested in the more pledgeable but lower return project. However, an increase in the pledgeability of the high return project tends to attract more investment to it, which is the case in our model only when the project returns are equal.

6 Extensions and Robustness

6.1 Risk Aversion

The benchmark model assumed that consumers are risk-neutral. We now show that resource allocation and equilibrium prices remain unchanged if consumers are risk averse. Specifically, let us assume that with probability q, the consumer is a late type with utility function $u(C_1 + C_2)$ whereas with probability $1-q$, the consumer's type is early with utility function $u(C_1)$. The function u satisfies the standard conditions: $u' > 0$ and $u'' \leq 0$. The rest of the model is unchanged.

The expected payoff of the consumer becomes

$$U = \max_{\theta} \, (1-q)u\left(\frac{\theta}{p_L}b_F + \frac{1-\theta}{p_S}\gamma_S R\right) + qu\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X\right).$$

An interior optimal θ leads to the following F.O.C.

$$(1-q)u'\left(\frac{\theta}{p_L}b_F + \frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{b_F}{p_L} - \frac{\gamma_S R}{p_S}\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S b_F}\gamma_L X\right) = 0.$$

If $b_F = \frac{q \frac{1-\theta}{p_S} \gamma_S R}{(1-q)\frac{\theta}{p_L}}$, this condition becomes

$$u'\left(\frac{1}{1-q}\frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{q(1-\theta)-(1-q)\theta}{\theta}\right)\frac{\gamma_S R}{p_S} = u'\left(\frac{1}{q}\frac{\theta}{p_L}\gamma_L X\right)\left(\frac{(1-q)\theta - q(1-\theta)}{(1-\theta)}\right)\frac{\gamma_L X}{p_L},$$

which only holds under $\theta = q$. If $b_F = \gamma_L X$, the F.O.C. becomes

$$(1-q)u'\left(\frac{\theta}{p_L}\gamma_L X + \frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{\gamma_L X}\gamma_L X\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) = 0,$$

which only holds under

$$\frac{\gamma_L X}{p_L} = \frac{\gamma_S R}{p_S}.$$

Therefore, introducing risk-aversion does not affect the consumer's resource allocation. Moreover, the rest of the equilibrium conditions are unchanged given that producers are still risk neutral.

6.2 Limited Transactability

We extend the analysis assuming that some buyers will be suspicious of or unwilling to trade with financial asset resellers. We assume a consumer can always sell a claim he has, but he can only buy with probability $\mu \in [0, 1]$, where we have assumed $\mu = 1$ thus far. What we have in mind is that the holder of a claim is informed about its value but the potential buyer of a claim need not be.[7] In a market with limited transparency and limited redress, only a fraction μ of buyers will turn out to be both informed and confident enough (for instance, because of their superior ability to seek redress) to purchase, the rest will be shut out of buying in the date-1 market. Let us term μ *transactability* – it can be both a property of the long term asset, as well as of market structure. Of course, since a lower μ thins out the buy side, it will (weakly) lower the sale price of the long asset, ensuring that buyers who are actually able to buy get better deals.

Compared to the case of $\mu = 1$, only the consumer's FOCs are different under $\mu < 1$. The analysis in the short glut equilibrium is unchanged, because the transactability of the long asset drops out of the consumer's FOC. Intuitively, the purchase price of the long asset at $t = 1$ is high enough to make the late consumer indifferent between purchasing (conditional on being able to transact) and simply consuming the purchase price directly. Therefore, transactability makes no difference to their utility in this region.

[7] Alternatively, the problem could be moral hazard. Suppose any seller could supply a worthless claim. Then only informed buyers will be willing to buy and worthless claims will not trade.

In the illiquid long with rent region, Equation (3) and (8) imply that the consumer's FOC becomes

$$\underbrace{q\mu\frac{1-\theta}{\theta}\frac{\gamma_S R}{p_S}}_{\text{long early}}+\underbrace{q\frac{\gamma_L X}{p_L}}_{\text{long late}}=\underbrace{(1-q)\frac{\gamma_S R}{p_S}}_{\text{short early}}+\underbrace{(1-q)\frac{\theta}{1-\theta}\frac{\gamma_L X}{p_L}}_{\text{short late access}}+\underbrace{q(1-\mu)\frac{\gamma_S R}{p_S}}_{\text{short late no access}}$$

$$\Rightarrow q\frac{\gamma_L X}{p_L}\left[\underbrace{1}_{\text{long late}}-\underbrace{\frac{1-q}{q}\frac{\theta}{1-\theta}}_{\text{short late access}}\right]=(1-q)\frac{\gamma_S R}{p_S}\left[\underbrace{1}_{\text{short early}}-\underbrace{\mu\frac{q}{1-q}\frac{1-\theta}{\theta}}_{\text{long early access}}+(1-\mu)\frac{q}{1-q}\right].$$

As we show in the appendix, the equilibrium solution can be captured as a a cubic equation in θ.

In the illiquid long no rent region, the consumer's F.O.C. continues to hold, implying

$$q\mu\frac{1-\theta}{\theta}\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}+qX=(1-q)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}+(1-q)\frac{\theta}{1-\theta}X+q(1-\mu)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}$$

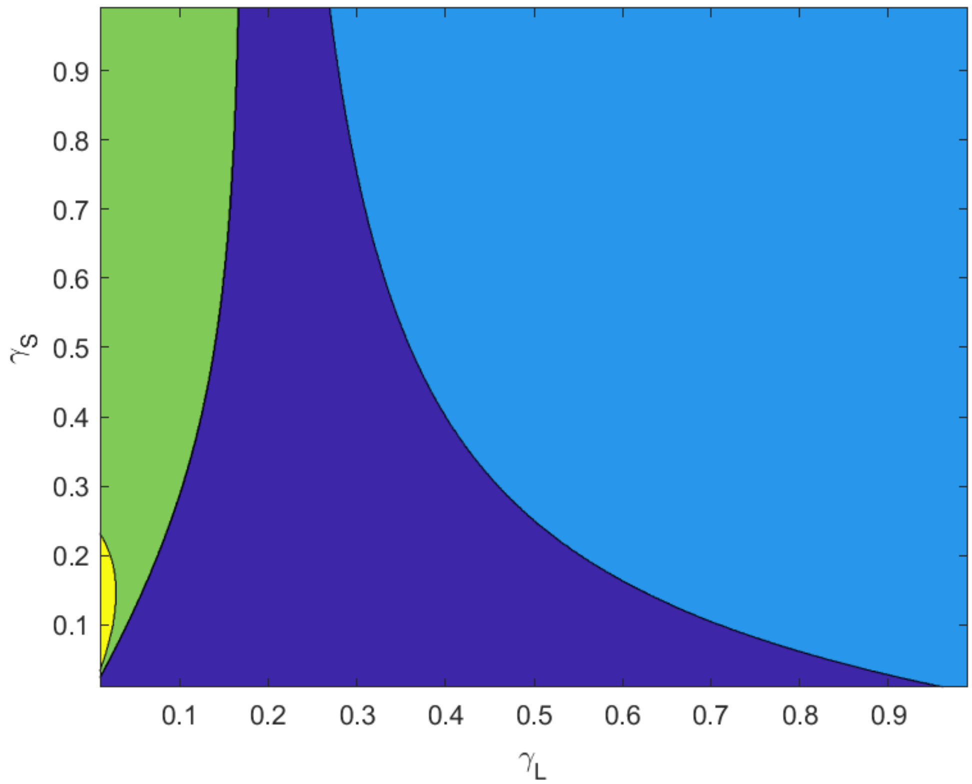
$$\Rightarrow q\mu\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}\left(\frac{1-\theta}{\theta}\right)^2$$

$$+\left(qX-(1-q)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}-q(1-\mu)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}\right)\left(\frac{1-\theta}{\theta}\right)-(1-q)X=0.$$

Finally in the short dominance region, μ is irrelevant because no long-term asset is produced.

Figure 8 plots the equilibrium regions as a function of γ_L and γ_S.

Figure 8: Equilibrium Cases a function of γ_L and γ_S



6.3 Planner's Problem

In this subsection, we examine benchmark financing, production, trading, and consumption decisions in the planner's problem. Throughout, we assume the social planner's objective function is to maximize

$$W = \alpha\lambda U + \Pi = \alpha\lambda \left(\underbrace{(1-q)C_1^E}_{\text{early type}} + \underbrace{q\left(C_1^L + C_2^L\right)}_{\text{late type}} \right) + (\Pi_1 + \Pi_2)\,,$$

where $\frac{\alpha}{1+\alpha}$ is the weight on consumers.We start with the first-best allocation and then move on to cases in which the planner faces different constraints.As we will see below, the first-best allocation and those under different constraints always yield a bang-bang solution whereby all the resources are either allocated to long- or short assets. Therefore, the decentralization outcome is never constrained-optimal.

We describe the allocation and leave the details to the appendix.

First-best allocation. The social planner wants no short asset produced since its return is dominated. Early consumers consume nothing since the consumer's expected utility is enhanced more for the same resource cost if late consumers consume (concave utility would change this stark assessment). Of course, depending on whose utility the social planner weighs more (that is, on α), either the consumer or the producer will consume.

Pledgeability-Constrained Allocation The pledgeability constraints require that the total consumption cannot exceed the pledgeable output produced by producers. These constraints alter how much can be promised to consumers out of the produced asset, and may tilt the social planner's preferences over which asset is produced, especially if consumers have high weight and the short pledgeability exceeds that of long.

Pledgeability- and Private Information-Constrained Allocation When the consumer type is private information, two additional constraints are needed to get types to select the consumption for their type: $C_1^E \geq C_1^L$ to get the early to self select and $C_1^L + C_2^L \geq C_1^E + C_2^E$ for the late. The allocation turns out to not be affected with the introduction of these additional constraints.

Pledgeability-, Private Information, and Producer Incentive-Constrained Allocation When the planner cannot set the total allocations to each asset, z_S and z_L, there is an incentive constraint on producers. Producers obtain all of the non-pledgeable part of any production. That is, only combinations of C_1 and C_2 that are no less profitable than others that the producer could produce are incentive compatible. In this case, the social planner's preferences over which asset is produced can be tilted if consumers have high weight and producers have conflicting preferences for production, that is, if $\alpha\gamma_S R + (1-\gamma_S)\, R > \alpha\gamma_L X + (1-\gamma_L)\, X$ and $(1-\gamma_S)R > (1-\gamma_L)X$ or $\alpha\gamma_S R + (1-\gamma_S)\, R < \alpha\gamma_L X + (1-\gamma_L)\, X$ and $(1-\gamma_S)R \leq (1-\gamma_L)X$. In the first case, the planner prefers the short asset whereas producers prefer long production, whereas the opposite holds in the second case. In both situations, more rents need to be offered to the producers.

7 Conclusion

This paper examines how financial development, through improved pledgeability of returns, affects production decisions and welfare in an economy with distinct producer and consumer groups. Our analysis yields several key insights that challenge conventional wisdom about the benefits of financial development.

We find that increased pledgeability does not always lead to higher output or welfare. In certain equilibrium regions, improving long-term asset pledgeability can actually reduce long-term production and overall welfare. The effects of financial development depend critically on the existing level of development and the relative scarcity of producer capital. As pledgeability improves, different equilibrium regimes emerge, each with distinct implications for production choices and welfare.

Our model implies important conflicts of interest over financial development between producers and consumers. Producers may oppose further development in intermediate stages, while consumers generally benefit from improved pledgeability. This dynamic helps explain why economies may face impediments to financial development and growth, especially when producer capital is scarce. Indeed, it can be interesting to extend the model and study the dynamics of consumer/producer capital and the production allocation.

We also find that credit development, characterized by improved short-term pledgeability, tends to benefit consumers but can harm producers. Its effects on overall welfare are more ambiguous, highlighting the complex interplay between different forms of financial development.

Interestingly, our results suggest that financial development becomes easier and faces less opposition at higher levels of development. This is partly because financing rents diminish and conflicts of interest abate as the economy progresses, creating a form of virtuous cycle in advanced stages of development.

These findings shed new light on the political economy of financial reform across different stages of development. They help explain why some economies may struggle to implement financial reforms or fall into development traps. Future research could explore how these dynamics play out in specific country contexts and examine policy interventions to overcome potential obstacles to financial development. By providing a more nuanced understanding of the complex relationships between pledgeability, production decisions, and welfare, this paper contributes to ongoing debates about the role of financial development in economic growth and may offer insights for policymakers navigating the challenges of financial reform.

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Appendix

Proof of Existence, Uniqueness, and Conditions for all cases

First, we prove equilibrium existence and uniqueness. Second, we establish conditions for the existence of each case. We express these conditions in terms of γ_L. As γ_L increases, the equilibrium type in general switches from dominance to short glut, followed by illiquid long with rent and finally illiquid long no rent. Depending on parameters, either the dominance or the illiquid long no rent case may not exist.

Dominance

1. The price $p_S = \frac{\lambda}{\lambda+1} \geq 1 - (1-\gamma_S)\frac{R}{X}$ implies that

$$(\lambda+1)(1-\gamma_S)R \geq X. \tag{10}$$

   Note that this condition is sufficient to guarantee that $p_S \geq \gamma_S$. This is a producer condition.

2. The condition of a shadow p_L requires

$$\begin{aligned}\frac{\gamma_L X}{\gamma_S R/p_S} &\leq 1 - \frac{(1-\gamma_L)X}{(1-\gamma_S)R}(1-p_S) \\ \gamma_L &\leq \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R - X}{\lambda-(\lambda+1)\gamma_S} \end{aligned} \tag{11}$$

   This is an equilibrium condition.

Short glut

We know from the Lemma that $X(1-\gamma_L) > R(1-\gamma_S)$ holds so that

$$\gamma_L < 1 - \frac{R}{X}(1-\gamma_S)$$

This condition implies $\gamma_L < \gamma_S$, $p_L < p_S$, and $\gamma_L X < \gamma_S R$. These results come from

$$\frac{(1-\gamma_L)X}{1-p_L} = \frac{(1-\gamma_S)R}{1-p_S}.$$

The condition $X(1-\gamma_L) > R(1-\gamma_S)$ implies $1-p_L > 1-p_S$ and equivalently $p_S > p_L$. Note that $\frac{p_S}{p_L} = \frac{\phi\gamma_S R}{\gamma_L X}$, so that $\gamma_L X < \phi\gamma_S R$. In the case of $\phi = 1$, then $\gamma_L X < \gamma_S R$. Given

$X > R$, it must be that $\gamma_L < \gamma_S$.

Moreover, we know

$$p_S = \frac{\gamma_S}{X} \frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L}$$
$$p_L = \frac{\gamma_L}{R} \frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L}.$$

1. $\theta = \frac{\frac{1}{\lambda} - \frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L} - \frac{1-p_S}{p_S}} \in [0,1]$ requires $\frac{1}{\lambda} \geq \frac{1-p_S}{p_S}$ and $\frac{1}{\lambda} \leq \frac{1-p_L}{p_L}$.

2. $y_L = \frac{\lambda\theta}{p_L}(1-p_L) \in [0,1]$, which requires $\frac{1}{\lambda} \geq \frac{1-p_L}{p_L}\theta$ and $p_L < 1$. The first condition becomes

$$\frac{1}{\lambda} \geq \frac{1-p_L}{p_L} \frac{\frac{1}{\lambda} - \frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L} - \frac{1-p_S}{p_S}}$$
$$\frac{1}{\lambda}\frac{1-p_L}{p_L} - \frac{1}{\lambda}\frac{1-p_S}{p_S} \geq \frac{1-p_L}{p_L}\frac{1}{\lambda} - \frac{1-p_L}{p_L}\frac{1-p_S}{p_S}$$
$$\Rightarrow \frac{1}{\lambda} \leq \frac{1-p_L}{p_L},$$

which is redundant given the first constraint. The second constraint becomes

$$\frac{\gamma_L}{R}\frac{X(1-\gamma_L) - R(1-\gamma_S)}{\gamma_S - \gamma_L} < 1$$
$$\Rightarrow \gamma_L X < \gamma_S R$$

which always holds under the lemma.

3. $p_L \geq \gamma_L$ (and $p_S \geq 1 - (1-\gamma_S)\frac{R}{X}$). The first simplifies into

$$(X-R)(1-\gamma_L) \geq 0,$$

which always holds. The second simplifies into

$$\gamma_S \geq \gamma_S - \gamma_L,$$

which also always holds.

4. $\frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}} \geq \gamma_L X$, which becomes $\theta \leq q$, which is stronger than the first condition

To summarize, beyond the conditions in the lemma ($\gamma_L < 1 - \frac{R}{X}(1-\gamma_S)$), we only need conditions such that $\theta \in [0, q]$, which becomes 1) $\frac{1}{\lambda} \geq \frac{1-p_S}{p_S}$ and 2) $\frac{1}{\lambda} \leq q\frac{1-p_L}{p_L} + (1-q)\frac{1-p_S}{p_S}$. We know

$$\begin{aligned}\frac{1-p_S}{p_S} &= \frac{1-\gamma_S}{\gamma_S}\frac{\gamma_S R - \gamma_L X}{X(1-\gamma_L) - R(1-\gamma_S)} \\ \frac{1-p_L}{p_L} &= \frac{1-\gamma_L}{\gamma_L}\frac{\gamma_S R - \gamma_L X}{X(1-\gamma_L) - R(1-\gamma_S)}\end{aligned}$$

The first becomes

$$\begin{aligned}\frac{1}{\lambda} &\geq \frac{(1-\gamma_S)(\gamma_S R - X\gamma_L)}{\gamma_S\left(X(1-\gamma_L) - R(1-\gamma_S)\right)} \\ \Rightarrow \gamma_S\left(X(1-\gamma_L) - R(1-\gamma_S)\right) &\geq \lambda(1-\gamma_S)(\gamma_S R - X\gamma_L) \\ \Rightarrow \gamma_S(X-R) + \gamma_S(\gamma_S R - \gamma_L X) &\geq \lambda(1-\gamma_S)(\gamma_S R - X\gamma_L) \\ \Rightarrow \gamma_S(X-R) &\geq \left[\lambda(1-\gamma_S) - \gamma_S\right](\gamma_S R - X\gamma_L).\end{aligned}$$

- If $\lambda(1-\gamma_S) - \gamma_S \leq 0 \Rightarrow \gamma_S \geq \frac{\lambda}{\lambda+1}$, this condition is redundant.
- If $\lambda(1-\gamma_S) - \gamma_S < 0 \Rightarrow \gamma_S < \frac{\lambda}{\lambda+1}$, then we need

$$\begin{aligned}\gamma_L &\geq \frac{\gamma_S}{X}\left(R - \frac{(X-R)}{\lambda - (\lambda+1)\gamma_S}\right) \\ \Rightarrow \gamma_L &\geq \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R - X}{\lambda - (\lambda+1)\gamma_S}\end{aligned}$$

  We can show this is less than $1 - \frac{R}{X}(1-\gamma_S)$. Note that if $(\lambda+1)(1-\gamma_S)R < X$ holds, so that (10) is violated, then the condition above is redundant.

The second condition becomes

$$\begin{aligned}\frac{1}{\lambda} &\leq q\frac{1-p_L}{p_L} + (1-q)\frac{1-p_S}{p_S} \\ \Rightarrow \frac{1}{\lambda} &\leq \left(q\frac{1-\gamma_L}{\gamma_L} + (1-q)\frac{1-\gamma_S}{\gamma_S}\right)\frac{\gamma_S R - \gamma_L X}{X(1-\gamma_L) - R(1-\gamma_S)} \\ \Rightarrow (X(1-\gamma_L) - R(1-\gamma_S)) &\leq \left(q\frac{1-\gamma_L}{\gamma_L} + (1-q)\frac{1-\gamma_S}{\gamma_S}\right)\lambda(\gamma_S R - \gamma_L X) \\ \Rightarrow (X(1-\gamma_L) - R(1-\gamma_S))\gamma_L\gamma_S &\leq \lambda\left(q(1-\gamma_L)\gamma_S + (1-q)(1-\gamma_S)\gamma_L\right)(\gamma_S R - \gamma_L X) \\ \Rightarrow & X\left(\lambda(1-q) - (\lambda+1)\gamma_S\right)\gamma_L^2 + \\ & \gamma_S\left(R(\lambda(q-1)-1) + \lambda q X + (\lambda+1)R\gamma_S + X\right)\gamma_L - qR\lambda\gamma_S^2 \leq 0\end{aligned}$$

We know the LHS is negative for $\gamma_L = 0$. If we evaluate the LHS at $\gamma_L = 1 - \frac{R}{X}(1-\gamma_S)$, we get

$$\frac{\lambda(X-R)\left(1-\gamma_S\right)\left((1-q)(X-R)+R\gamma_S\right)}{X}>0.$$

If we evaluate the LHS at $\gamma_L=\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}$, we get

$$-\frac{\lambda(\lambda+1)q(X-r)^2\left(1-\gamma_S\right)\gamma_S^2}{X\left(\lambda-(\lambda+1)\gamma_S\right)^2}<0.$$

To summarize. Define $\underline{\gamma}_L \in \left(\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, 1-\frac{R}{X}(1-\gamma_S)\right)$ be the unique root that solves

$$X\left(\lambda(1-q)-(\lambda+1)\gamma_S\right)\gamma_L^2+\gamma_S\left(R(\lambda(q-1)-1)+\lambda qX+(\lambda+1)R\gamma_S+X\right)\gamma_L-qR\lambda\gamma_S^2=0,$$

- If $(\lambda+1)(1-\gamma_S)R<X$, then we need

$$\gamma_L\in[0,\underline{\gamma}_L]. \tag{12}$$

- Otherwise, we need

$$\gamma_L\in\left[\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S},\underline{\gamma}_L\right]. \tag{13}$$

Illiquid long with rent

We can show that the equilibrium reduces to a quadratic equation on y_L:

$$\begin{aligned}\left(X(1-\gamma_L)-R(1-\gamma_S)\right)y_L^2+\\ \left[\lambda\left(qX(1-\gamma_L)+(1-q)R(1-\gamma_S)\right)-\left(X(1-\gamma_L)-R(1-\gamma_S)\right)\right]y_L-\lambda qX(1-\gamma_L)=0.\end{aligned}$$

In equilibrium, both $(1-\gamma_L)X>(1-\gamma_S)R$ and $(1-\gamma_L)X<(1-\gamma_S)R$ can hold. In the first case, $p_L<p_S$, and $y_L>q$. In the second case, $p_L>p_S$, and $y_L<q$. By evaluating the LHS of the above equation, we know that the value is negative at $y_L=0$. At $y_L=1$, the value is

$$\lambda(1-q)R(1-\gamma_S)>0.$$

Therefore, there exists a unique y_L that solves this equation.

1. $y_L\in[0,1]$. This is obviously satisfied.

2. $\theta=q\in[0,1]$ is always satisfied

3. $b_F=\frac{p_L\gamma_S R}{p_S}\leq\gamma_L X$; $p_L=\frac{q\lambda}{(q\lambda+y_L)}$ and $p_S=\frac{\lambda(1-q)}{\lambda(1-q)+(1-y_L)}$. The condition $b_F=\frac{p_L\gamma_S R}{p_S}\leq$

$\gamma_L X$ simplifies into

$$y_L \geq \frac{q\lambda\left[\gamma_S R\left(\lambda(1-q)+1\right)-(1-q)\lambda\gamma_L X\right]}{(1-q)\lambda\gamma_L X+q\lambda\gamma_S R}.$$

- If $\gamma_S R\left(\lambda(1-q)+1\right)-(1-q)\lambda\gamma_L X<0 \Rightarrow \gamma_L>\frac{\gamma_S R((\lambda-\lambda q)+1)}{X(\lambda-\lambda q)}$ so that the RHS is negative, this condition is redundant.
- If $\gamma_S R\left(\lambda(1-q)+1\right)-(1-q)\lambda\gamma_L X>0$, then there are two cases:
  - If $X(1-\gamma_L)-R(1-\gamma_S)>0$, then we need to plug in $\frac{q\lambda[\gamma_S R(\lambda(1-q)+1)-(1-q)\lambda\gamma_L X]}{(1-q)\lambda\gamma_L X+q\lambda\gamma_S R}$ into the equation and the resulting number is negative.
  - If $X(1-\gamma_L)-R(1-\gamma_S)\leq 0$, then we also need to plug in $\frac{q\lambda[\gamma_S R(\lambda(1-q)+1)-(1-q)\lambda\gamma_L X]}{(1-q)\lambda\gamma_L X+q\lambda\gamma_S R}$ into the equation and the resulting number is negative.
  - In both cases, when we plug in, we get the sign is equal to the sign of

    $$-\left\{\gamma_L\gamma_S\left(R(\lambda(q-1)-1)+\lambda qX+(\lambda+1)R\gamma_S+X\right)-X\gamma_L^2\left(\lambda(q-1)+(\lambda+1)\gamma_S\right)-\lambda qR\gamma_S^2\right\},$$

    which is the same one as the short glut case. In order for this to be negative, we need

    $$\gamma_L\gamma_S\left(R(\lambda(q-1)-1)+\lambda qX+(\lambda+1)R\gamma_S+X\right)-X\gamma_L^2\left(\lambda(q-1)+(\lambda+1)\gamma_S\right)-\lambda qR\gamma_S^2>0,$$

    which requires $\gamma_L \geq \underline{\gamma}_L$.
- Combining the previous two cases, all we need for this case is to have $\gamma_L \geq \min\{\underline{\gamma}_L, \frac{\gamma_S R((\lambda-\lambda q)+1)}{X(\lambda-\lambda q)}\}$. We evaluate the LHS of the equation above at $\frac{\gamma_S R((\lambda-\lambda q)+1)}{X(\lambda-\lambda q)}$ and the sign is the same as

  $$\lambda(1-q)X-\left(\lambda(1-q)+1\right)\gamma_S R.$$

  We know that the above equation is positive whenever $\gamma_S R\left(\lambda(1-q)+1\right)-(1-q)\lambda\gamma_L X<0$, which implies $\underline{\gamma}_L=\min\{\underline{\gamma}_L, \frac{\gamma_S R((\lambda-\lambda q)+1)}{X(\lambda-\lambda q)}\}$. Therefore, this case needs $\gamma_L \geq \underline{\gamma}_L$.

4. $p_S \geq 1-(1-\gamma_S)\frac{R}{X}$, $p_L \geq \gamma_L$. The two conditions become:

$$y_L \geq \frac{X-(1-\gamma_S)R\left[\lambda(1-q)+1\right]}{X-(1-\gamma_S)R}$$

and

$$\frac{q\lambda}{q\lambda+y_L} \geq \gamma_L \Rightarrow y_L \leq q\lambda\frac{1-\gamma_L}{\gamma_L}.$$

When we evaluate the LHS of the equation at $\frac{X-(1-\gamma_S)R[\lambda(1-q)+1]}{X-(1-\gamma_S)R}$, we need it to be negative. When we evaluate the LHS of the equation at $q\lambda\frac{1-\gamma_L}{\gamma_L}$, we need it to be positive. It turns out that both equations reduce to

$$\lambda q\,(X-R(1-\gamma_S)) > \gamma_L\,((1+\lambda q)X-(\lambda+1)(1-\gamma_S)R)$$
$$\Rightarrow \gamma_L < \frac{\lambda q\,(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}.$$

- If $X-(1-\gamma_S)R\,[\lambda(1-q)+1]<0$, the first condition is not needed, and $\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R} > 1$. In this case, no further condition is needed.
- If $X-(1-\gamma_S)R\,[\lambda(1-q)+1]\geq 0$, then we need $\gamma_L < \frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}$.

To summarize, this case needs

$$\gamma_L > \underline{\gamma}_L. \tag{14}$$

If in addition,

$$(\lambda(1-q)+1)\,(1-\gamma_S)R < X \tag{15}$$

this case also needs

$$\gamma_L < \frac{\lambda q\,(X-(1-\gamma_S)R)}{(1+\lambda q+1)-(\lambda+1)(1-\gamma_S)R}. \tag{16}$$

Illiquid long no rent

We know in equilibrium $\theta = q$, $y_L = \frac{\lambda q(1-\gamma_L)}{\gamma_L}$, $y_S = \frac{\lambda(1-q)}{1-(1-\gamma_S)\frac{R}{X}}(1-\gamma_S)\frac{R}{X}$ and $b_F = \frac{\gamma_L\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}$.

1. $\theta\in[0,1]$ is always guaranteed.
2. $b_F \leq \gamma_L X$ can be shown simplified into $R\leq X$ so always holds.
3. $y_S\in[0,1]$, $y_L\in[0,1]$ and $y_S+y_L\in[0,1]$. $y_S\in[0,1]$ becomes

$$(\lambda(1-q)+1)\,(1-\gamma_S)R < X. \tag{17}$$

Note this condition does not require γ_L. $y_L\in[0,1]$ is less stringent than $y_L\leq 1-y_S$, which becomes

$$\gamma_L > \frac{\lambda q}{\lambda q+1-\frac{\lambda(1-q)}{1-(1-\gamma_S)\frac{R}{X}}(1-\gamma_S)\frac{R}{X}} = \frac{\lambda q\,(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}. \tag{18}$$

Summarizing Conditions for All Cases

1. $(\lambda(1-q)+1)(1-\gamma_S)R < X \leq (\lambda+1)(1-\gamma_S)R$. All four regions exist

   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}]$: short dominance

   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, \underline{\gamma}_L]$: short glut

   (c) $\gamma_L \in [\underline{\gamma}_L, \frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}]$: illiquid long with rent

   (d) $\gamma_L \in [\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}, 1]$: illiquid long no rent

2. $X > (\lambda+1)(1-\gamma_S)R$. There is no short dominance region

   (a) $\gamma_L \in [0, \underline{\gamma}_L]$: short glut

   (b) $\gamma_L \in [\underline{\gamma}_L, \frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}]$: illiquid long with rent

   (c) $\gamma_L \in [\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}, 1]$: illiquid long no rent

3. $X < (\lambda(1-q)+1)(1-\gamma_S)R$. There is no illiquid long no rent region

   (a) $\gamma_L \in [0, \frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}]$: short dominance

   (b) $\gamma_L \in [\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}, \underline{\gamma}_L]$: short glut

   (c) $\gamma_L \in [\underline{\gamma}_L, 1]$: illiquid long with rent.

Comparative Statics with respect to γ_L

Proof of Lemma 4

Proof. We know that

$$\frac{\partial p_S}{\partial \gamma_L} = \frac{-(1-\gamma_S)\gamma_S(R-X)}{X(\gamma_L-\gamma_S)^2}.$$

Given $R-X<0$, we know that

$$\frac{\partial p_S}{\partial \gamma_L} > 0.$$

Because$\frac{p_L}{p_S} = \frac{\gamma_L X}{\gamma_S R}$, this immediately implies that $\frac{\partial p_L}{\partial \gamma_L} > 0$, and also p_L must increase more than proportionately with γ_L for the equality to hold, so that $\frac{\partial(\gamma_L/p_L)}{\partial \gamma_L} < 0$. Given that

$$\theta = \frac{\frac{1}{\lambda} - \frac{1-p_S}{p_S}}{\frac{1-p_L}{p_L} - \frac{1-p_S}{p_S}} \in (0,1),$$

we know that if p_L stays unchanged, the RHS would increase in γ_L. Now that $\frac{1-p_L}{p_L}$ decreases with γ_L, we know θ must increase in γ_L. The market clearing condition implies

$$y_S = \frac{\lambda(1-\theta)(1-p_S)}{p_S}$$

must decrease in γ_L, implying that y_L increases in γ_L.

Both sides of the producer's equilibrium condition

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

go up with γ_L, given that p_S increases. Therefore, producer's profits Π increases with γ_L. We know that consumer welfare is

$$U = \frac{\gamma_S R}{p_S}$$

which decreases with γ_L. Finally, turning to total welfare. We can write

$$W = (\lambda\theta + y_L)X + (\lambda(1-\theta) + (1-y_L))R,$$

which increases in γ_L given both y_L and θ increase in γ_L. □

Proof of Lemma 5

Proof. (i) From

$$\frac{(1-\gamma_L)X}{\lambda(1-q)+(1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\lambda q + y_L}y_L,$$

we know that when γ_L goes up, producer investment in the long asset, y_L, must go down. Here is why: If y_L goes up, the producer's first order condition cannot hold. Intuitively, an increase in long pledgeability$\gamma_L$would, ceteris paribus, reduce the producer's incentive to invest in the long asset below the short asset. To restore producer incentives, it must be that p_L increases, which can only be if the producer invests less in the long asset (since consumer allocations do not change), that is, y_L falls.

Consequently, $p_S(=\frac{\lambda(1-q)}{\lambda(1-q)+(1-y_L)})$ falls with γ_L so that $\frac{\gamma_S R}{p_S}$ increases with γ_L. Now, let us turn to $\frac{\gamma_L X}{p_L}$. We are going to show this also increases. If p_S goes down with γ_L,the producer's cum-financing return on the short asset falls (the LHS of the producer's FOC below), so the cum-financing return on the long asset should also fall (the RHS of the FOC below).

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

This implies

$$\frac{d\frac{(1-\gamma_L)}{1-p_L}}{d\gamma_L} < 0 \Rightarrow -(1-p_L) + (1-\gamma_L)\frac{dp_L}{d\gamma_L} < 0 \Rightarrow \frac{dp_L}{d\gamma_L} < \frac{1-p_L}{1-\gamma_L}.$$

Meanwhile,

$$\frac{d\frac{\gamma_L}{p_L}}{d\gamma_L} = \frac{p_L - \gamma_L \frac{dp_L}{d\gamma_L}}{\gamma_L^2} > \frac{p_L - \gamma_L \frac{1-p_L}{1-\gamma_L}}{\gamma_L^2} > \frac{\frac{p_L}{\gamma_L} - \frac{1-p_L}{1-\gamma_L}}{\gamma_L} > 0.$$

The last inequality holds because $p_L \geq \gamma_L$. Therefore, both $\frac{\gamma_S R}{p_S}$ and $\frac{\gamma_L X}{p_L}$, the consumer's hold to maturity returns, increase with γ_L.

(ii) Consumer welfare i is given by $U = (1-q)\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L}$, which clearly increases in consumer returns $\frac{\gamma_S R}{p_S}$ and $\frac{\gamma_L X}{p_L}$, and hence increases with γ_L.

Turning to producer profits: $\Pi = \frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$,which we have seen falls in γ_L since the cum financing producer returns fall on either asset. Finally, total welfare (assuming equal weights) is just total production since there are no frictions in trade, which is

$$\lambda U + \Pi = X y_L + R(1-y_L) + \lambda(qX + (1-q)R),$$

which increases in y_L, and hence falls in γ_L. □

Proof of Lemma 6

Proof. The other expressions are obvious. We supplement the expressions for welfare here. consumer welfare is

$$U = (1-q)\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L} = (1-q)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}} + qX.$$

Producer profits are

$$\Pi = X.$$

□

Comparative Statics with respect to γ_S

Proof of Lemma 7

Proof. From

$$\frac{(1-\gamma_L)X}{\lambda(1-q)+(1-y_L)}(1-y_L) = \frac{(1-\gamma_S)R}{\lambda q + y_L}y_L,$$

we know that when γ_S goes up, y_L must go up. If y_L goes down, the RHS goes down, whereas the LHS goes up. The equation cannot hold. – the producer shifts towards long production since short production has become unattractive at the old prices. Given this result, the total welfare $\lambda U + \Pi$ goes up. Also $p_L = \frac{q\lambda}{q\lambda + y_L}$ goes down and $p_S = \frac{\lambda(1-q)}{\lambda(1-q)+(1-y_L)}$ goes up. Coming to consumer welfare

$$U = (1-q)\,\frac{\gamma_S R}{p_S} + q\frac{\gamma_L X}{p_L}.$$

Clearly, $\frac{\gamma_L X}{p_L}$ goes up. We show $\frac{\gamma_S R}{p_S}$ also goes up. Specifically, we know

$$\frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

both go down. This implies

$$\frac{d\frac{(1-\gamma_S)}{1-p_S}}{d\gamma_S} < 0 \Rightarrow -(1-p_S) + (1-\gamma_S)\frac{dp_S}{d\gamma_S} < 0 \Rightarrow \frac{dp_S}{d\gamma_S} < \frac{1-p_S}{1-\gamma_S}.$$

Meanwhile,

$$\frac{d\frac{\gamma_S}{p_S}}{d\gamma_S} = \frac{p_S - \gamma_S\frac{dp_S}{d\gamma_S}}{\gamma_S^2} > \frac{p_S - \gamma_S\frac{1-p_S}{1-\gamma_S}}{\gamma_S^2} > \frac{\frac{p_S}{\gamma_S} - \frac{1-p_S}{1-\gamma_S}}{\gamma_S} > 0.$$

The last inequality holds because $p_S > \gamma_S$. Therefore, consumer welfare goes up. Finally, producer profits are:

$$\Pi = \frac{(1-\gamma_S)R}{1-p_S} = \frac{(1-\gamma_L)X}{1-p_L}$$

Given that p_L goes down, Π also goes down. □

Proof of Lemma 8

Proof. We know that

$$\frac{\partial p_L}{\partial \gamma_S} = -\frac{(1-\gamma_L)\,\gamma_L(X-R)}{R\,(\gamma_L-\gamma_S)^{\,2}} < 0.$$

Therefore, $\frac{\gamma_L X}{p_L}$ goes up, which implies $\frac{\gamma_S R}{p_S}$ also goes up. consumer welfare

$$U = \frac{\gamma_L X}{p_L}$$

goes up. Producer's profits

$$\Pi = \frac{(1-\gamma_L)X}{1-p_L}$$

go down. $\square$

Comparative Statics with respect to λ

We supplement the analysis on how the thresholds in γ_L for different regions vary. By taking first-order derivatives, it is easily verified that both $\frac{\gamma_S}{X}\frac{(\lambda+1)(1-\gamma_S)R-X}{\lambda-(\lambda+1)\gamma_S}$ and $\frac{\lambda q(X-(1-\gamma_S)R)}{(1+\lambda q)X-(\lambda+1)(1-\gamma_S)R}$ increase with λ. To study $\underline{\gamma}_L$, let us rewrite the equation that solves $\underline{\gamma}_L$:

$$X\left(\lambda(1-q)-(\lambda+1)\gamma_S\right)\gamma_L^2+\gamma_S\left(R(\lambda(q-1)-1)+\lambda qX+(\lambda+1)R\gamma_S+X\right)\gamma_L-qR\lambda\gamma_S^2$$
$$\lambda\left\{X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2\right\}+X\left(-\gamma_S\right)\gamma_L^2+\gamma_S\left(-R+R\gamma_S+X\right)\gamma_L$$
$$\lambda\left\{X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2\right\}+\left[X\left(1-\gamma_L\right)-R(1-\gamma_S)\right]\gamma_S\gamma_L$$

Given that $X\left(1-\gamma_L\right)-R(1-\gamma_S)>0$ holds on $(\underline{\gamma}_L-\varepsilon,\underline{\gamma}_L+\varepsilon)$ for ε sufficiently small, we know that the coefficient in front of λ must satisfy

$$X\left((1-q)-\gamma_S\right)\gamma_L^2+\gamma_S\left(R((q-1))+qX+R\gamma_S\right)\gamma_L-qR\gamma_S^2<0.$$

Therefore, the solution $\underline{\gamma}_L$ must increase in λ.

Proof of Lemma 11

Proof. We can rewrite the equation that determines y_L as

$$(1-\gamma_L)X\frac{1-y_L}{\lambda(1-q)+(1-y_L)}=(1-\gamma_S)R\frac{y_L}{\lambda q+y_L}$$
$$\Rightarrow\frac{1+\lambda q/y_L}{1+\lambda(1-q)/(1-y_L)}=\frac{(1-\gamma_S)R}{(1-\gamma_L)X}.$$

We differentiate both sides and get:

$$\underbrace{\left[\lambda\frac{1-q}{(1-y_L)^2}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}+\lambda\frac{q}{y_L^2}\right]}_{>0}\frac{dy_L}{d\lambda}=\frac{q}{y_L}-\frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}.$$

Therefore, the sign of $\frac{dy_L}{d\lambda}$ depends on the sign of $\frac{q}{y_L} - \frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}$. Clearly,

$$\begin{aligned}\text{sign}\left(\frac{q}{y_L} - \frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}\right) &= \text{sign}\left(\frac{\lambda q}{y_L} - \lambda\frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X}\right) \\ &= \text{sign}\left(\frac{(1-\gamma_S)R}{(1-\gamma_L)X} - 1\right),\end{aligned}$$

where the last inequality follows from

$$\begin{aligned}\frac{1+\lambda q/y_L}{1+\lambda(1-q)/(1-y_L)} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} \\ \Rightarrow 1 + \frac{\lambda q}{y_L} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} + \frac{(1-\gamma_S)R}{(1-\gamma_L)X}\lambda(1-q)/(1-y_L) \\ \Rightarrow \frac{\lambda q}{y_L} - \lambda\frac{1-q}{1-y_L}\frac{(1-\gamma_S)R}{(1-\gamma_L)X} &= \frac{(1-\gamma_S)R}{(1-\gamma_L)X} - 1.\end{aligned}$$

□

Detailed Analysis

Illiquid long no rent

The consumer's F.O.C. continues to hold, implying

$$\begin{aligned}& q\frac{1-\theta}{\theta}\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}} + qX = (1-q)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}} + (1-q)\frac{\theta}{1-\theta}X \\ \Rightarrow\ & q\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}\left(\frac{1-\theta}{\theta}\right)^2 + \left(qX - (1-q)\frac{\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}\right)\left(\frac{1-\theta}{\theta}\right) - (1-q)X = 0,\end{aligned}$$

which only admits one positive root to $\frac{1-\theta}{\theta}$. Clearly, $\theta = q$. The market clearing for short long (4) and (5) determine the allocation choices of producers. The rest of the solutions are

$$\begin{aligned}\lambda\frac{\theta}{p_L} &= \frac{y_L}{1-p_L} \Rightarrow y_L = \frac{\lambda q(1-\gamma_L)}{\gamma_L} \\ \lambda\frac{1-\theta}{p_S} &= \frac{y_S}{1-p_S} \Rightarrow y_S = \frac{\lambda(1-q)}{1-(1-\gamma_S)\frac{R}{X}}(1-\gamma_S)\frac{R}{X} \\ b_F &= \frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}} = \frac{p_L\gamma_S R}{p_S} = \frac{\gamma_L\gamma_S R}{1-(1-\gamma_S)\frac{R}{X}}.\end{aligned}$$

The conditions for equilibrium is: 1) $\theta \in [0,1]$; 2) $b_F \leq \gamma_L X$, 3) $y_S \in [0,1]$, $y_L \in [0,1]$, and $y_S + y_L \leq 1$.

Limited Transactionability

Here we provide the details analysis of the market pricing case under limited transactionability $\mu < 0$. Let us being by listing the system of equations

$$\frac{(1-\gamma_S)\,R}{1-p_S} = \frac{(1-\gamma_L)\,X}{1-p_L}$$
$$\Rightarrow q\frac{\gamma_L X}{p_L}\left(1-\frac{1-q}{q}\frac{\theta}{1-\theta}\right) = (1-q)\,\frac{\gamma_S R}{p_S}\left[1-\mu\frac{q}{1-q}\frac{1-\theta}{\theta}+(1-\mu)\,\frac{q}{1-q}\phi\right]$$
$$\theta\frac{1-p_L}{p_L}+(1-\theta)\,\frac{1-p_S}{p_S} = \frac{1}{\lambda}.$$

Now, we show that this reduces to a cubic one on θ. Specifically, let $\hat{z} = \frac{1-q}{q}\frac{\theta}{1-\theta}$ and $z = \frac{\theta}{1-\theta} = \frac{q}{1-q}\hat{z} \Rightarrow \theta = \frac{z}{z+1}$, $1-\theta = \frac{1}{z+1}$. The middle equation becomes

$$\frac{p_L}{A} = \frac{p_S}{B},$$

where

$$A = A_1 - A_2 z$$
$$A_1 = q\gamma_L X$$
$$A_2 = (1-q)\gamma_L X$$
$$B = B_1 - \frac{B_2}{z}$$
$$B_1 = (1-q)\gamma_S R\left(1+(1-\mu)\frac{q}{1-q}\phi\right)$$
$$B_2 = q\gamma_S R\mu.$$

The first equation becomes

$$\frac{1-p_S}{C} = \frac{1-p_L}{D}$$
$$C = (1-\gamma_S)R$$
$$D = (1-\gamma_L)X.$$

From here, we get

$$p_S = \frac{D-C}{D-C\frac{A}{B}} \Rightarrow \frac{1-p_S}{p_S} = \frac{C-C\frac{A}{B}}{D-C}$$
$$p_L = \frac{D-C}{\frac{B}{A}D-C} \Rightarrow \frac{1-p_L}{p_L} = \frac{\frac{B}{A}D-D}{D-C}.$$

The cubic equation is

$$\begin{aligned}
&\left(-A_2^2C + A_2B_1D - \frac{A_2B_1(C-D)}{\lambda}\right)z^3 \\
&+\left(A_2(2A_1-B_1)C + (-A_1B_1+B_1^2-A_2B_2)D + \frac{[A_1B_1+A_2(B_2-B_1)]\,(C-D)}{\lambda}\right)z^2 \\
&+\left(-A_1^2C + A_1B_1C + A_2B_2C + A_1B_2D - 2B_1B_2D + \frac{[A_1(B_1-B_2)+A_2B_2]\,(C-D)}{\lambda}\right)z \\
&-A_1B_2C + B_2^2D - \frac{A_1B_2(C-D)}{\lambda} = 0
\end{aligned}$$

If it occurs that $(1-\gamma_S)\,R = (1-\gamma_L)\,X$, then we immediately have

$$p_L = p_S = \frac{\lambda}{1+\lambda}.$$

In this case, let $\hat{z} = \frac{1-q}{q}\frac{\theta}{1-\theta}$, the middle equation becomes

$$\begin{aligned}
&q\gamma_L X\,(1-\hat{z}) = (1-q)\,\gamma_S R\left[1-\mu\frac{1}{\hat{z}} + (1-\mu)\,\frac{q}{1-q}\phi\right] \\
&\Rightarrow q\gamma_L X\hat{z}^2 - \left(q\gamma_L X - (1-q)\,\gamma_S R\left[1+(1-\mu)\,\frac{q}{1-q}\phi\right]\right)\hat{z} - (1-q)\,\gamma_S R\mu = 0
\end{aligned}$$

Risk Aversion

We show that resource allocation and equilibrium prices, remain unchanged if consumers are risk averse. Specifically, let us assume that with probability q, the consumer is a late type with utility function $u(C_1+C_2)$ whereas with probability $1-q$, the consumer's type is early with utility function $u(C_1)$. The function u satisfies the standard conditions: $u' > 0$ and $u'' \leq 0$. The rest of the model is unchanged.

The expected payoff of the consumer becomes

$$U = \max_{\theta}\,(1-q)u\left(\frac{\theta}{p_L}b_F + \frac{1-\theta}{p_S}\gamma_S R\right) + qu\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X\right).$$

We first rule out the corner solution $\theta = 1$: if $\theta = 1$, then $b_F = 0$, and $\frac{\partial U}{\partial \theta} \to -\infty$, violating that $\theta = 1$ is optimal. An interior optimal θ leads to the following F.O.C.

$$(1-q)u'\left(\frac{\theta}{p_L}b_F + \frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{b_F}{p_L} - \frac{\gamma_S R}{p_S}\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S b_F}\gamma_L X\right) = 0.$$

If $b_F = \frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}} \leq \gamma_L X$, the F.O.C. gets simplified to

$$(1-q)u'\left(\frac{1}{1-q}\frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{q(1-\theta)}{(1-q)\theta} - 1\right)\frac{\gamma_S R}{p_S} + qu'\left(\frac{1}{q}\frac{\theta}{p_L}\gamma_L X\right)\left(1 - \frac{(1-q)\theta}{q(1-\theta)}\right)\frac{\gamma_L X}{p_L} = 0$$

$$\Rightarrow u'\left(\frac{1}{1-q}\frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{q(1-\theta)-(1-q)\theta}{\theta}\right)\frac{\gamma_S R}{p_S} = u'\left(\frac{1}{q}\frac{\theta}{p_L}\gamma_L X\right)\left(\frac{(1-q)\theta - q(1-\theta)}{(1-\theta)}\right)\frac{\gamma_L X}{p_L},$$

where the only solution is $\theta = q$. Otherwise, we will have

$$\Rightarrow u'\left(\frac{1}{1-q}\frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{1}{\theta}\right)\frac{\gamma_S R}{p_S} = -u'\left(\frac{1}{q}\frac{\theta}{p_L}\gamma_L X\right)\left(\frac{1}{(1-\theta)}\right)\frac{\gamma_L X}{p_L},$$

which can never hold.

If $b_F = \gamma_L X \leq \frac{q\frac{1-\theta}{p_S}\gamma_S R}{(1-q)\frac{\theta}{p_L}}$ instead, the F.O.C. gets simplified to

$$(1-q)u'\left(\frac{\theta}{p_L}\gamma_L X + \frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{\gamma_L X}\gamma_L X\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) = 0.$$

Again, this can only hold if

$$\frac{\gamma_L X}{p_L} = \frac{\gamma_S R}{p_S}.$$

Otherwise, we have

$$(1-q)u'\left(\frac{\theta}{p_L}\gamma_L X + \frac{1-\theta}{p_S}\gamma_S R\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{\gamma_L X}\gamma_L X\right) = 0,$$

which can never hold.

$$(1-q)u'\left(\frac{\theta}{p_L}b_F + \frac{1-\theta}{p_S}\gamma_S R\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) + qu'\left(\frac{\theta}{p_L}\gamma_L X + \frac{\frac{1-\theta}{p_S}\gamma_S R}{b_F}\gamma_L X\right)\left(\frac{\gamma_L X}{p_L} - \frac{\gamma_S R}{p_S}\right) < 0$$

Therefore, introducing risk-aversion does not affect the consumer's resource allocation. Moreover, the rest of the equilibrium conditions are unchanged given that producers are still risk neutral. Therefore, we can conclude that resource allocation and equilibrium prices, remain unchanged.

Social Planner's Problem

First-best allocation

Let us assume the social-welfare function takes the form of.

$$W = \alpha\lambda U + \Pi = \alpha\lambda \left(\underbrace{(1-q)C_1^E}_{\text{early type}} + \underbrace{q\left(C_1^L + C_2^L\right)}_{\text{late type}} \right) + (\Pi_1 + \Pi_2)$$

where $\frac{\alpha}{1+\alpha}$ is the positive weight on consumers. Implicitly, we assume the welfare function has equal weights within consumers. The resource constraint is

$$\frac{\lambda(1-q)C_1^E}{R} + \frac{\lambda q C_1^L}{R} + \frac{\lambda q C_2^L}{X} + \frac{\Pi_1}{R} + \frac{\Pi_2}{X} = \lambda + 1.$$

Our next result describes the first-best allocation.

Lemma 13. *In the first-best allocation, it is without loss of generality to let* $C_1^L = 0$, $C_1^E = 0$ *and* $\Pi_1 = 0$. *Moreover,*

1. If $\alpha > 1$, then $\Pi_2 = 0$, and $C_2^L = \frac{(\lambda+1)X}{\lambda q}$.
2. If $\alpha < 1$, then $C_2^L = 0$, and $\Pi_2 = X(\lambda + 1)$.
3. If $\alpha = 1$, then any combination of C_2^L and Π_2 that satisfies $\frac{\lambda q C_2^L}{X} + \frac{\Pi_2}{X} = \lambda + 1$ attains first-best allocation.

Proof. See Appendix □

In the unconstrained problem, the social planner wants no short asset produced since its return is dominated. So early consumers consume nothing since the consumer's expected utility is enhanced more for the same resource cost if late consumers consume (concave utility would change this stark assessment). Of course, depending on whose utility the social planner weighs more (that is, on α), either the consumer or the producer will consume. This allocation clearly does not take into account either pledgeability constraints (how much of the asset's returns can be allocated to consumers) or property rights (who has capital up front or assets at $t = 1$).

Pledgeability-Constrained Allocation

Let us now add the constraints on pledgeability (we do not take into account who owns the capital up front at $t = 0$). Let z_S and z_L be the total resources allocated to short and long-term production at $t = 0$. Clearly, we have $z_S + z_L = \lambda + 1$. Moreover, the pledgeability constraint implies that consumer's consumption on both dates are constrained by the pledgeable cash flows generated from the assets, i.e.

$$\begin{aligned} \lambda(1-q)C_1^E + \lambda q C_1^L &\leq z_S \gamma_S R \\ \lambda q C_2^L &\leq z_L \gamma_L X, \end{aligned}$$

and producers' profits are bounded below by the non-pledgeable cash flows from producing the two types of assets

$$\begin{aligned} z_S R \geq \Pi_1 &\geq z_S(1-\gamma_S)R \\ z_L X \geq \Pi_2 &\geq z_L(1-\gamma_L)X. \end{aligned}$$

Finally, we introduce the resource constraints at both $t = 1$ and $t = 2$

$$\begin{aligned} \lambda(1-q)C_1^E + \lambda q C_1^L + \Pi_1 &= z_S R \\ \lambda q C_2^L + \Pi_2 &= z_L X. \end{aligned}$$

Our next result summarizes the pledgeability constrained-optimal allocation.

Lemma 14. *In the pledgeability constrained-optimal allocation, we have*

1. If $\alpha > 1$,
    - If $\alpha\gamma_S R + (1-\gamma_S) R > \alpha\gamma_L X + (1-\gamma_L) X$, then $z_S = \lambda + 1$ and $z_L = 0$. In this case, $\lambda(1-q)C_1^E + \lambda q C_1^L = (\lambda+1)\gamma_S R$, $C_2^L = 0$, $\Pi_1 = (\lambda+1)(1-\gamma_S) R$, and $\Pi_2 = 0$.
    - If $\alpha\gamma_S R + (1-\gamma_S) R < \alpha\gamma_L X + (1-\gamma_L) X$, then $z_S = 0$ and $z_L = \lambda + 1$. In this case, $C_1^E = C_1^L = 0$, $C_2^L = \frac{(\lambda+1)\gamma_L X}{\lambda q}$, $\Pi_1 = 0$, and $\Pi_2 = (\lambda+1)(1-\gamma_L) X$.
    - If $\alpha\gamma_S R + (1-\gamma_S) R = \alpha\gamma_L X + (1-\gamma_L) X$, then any z_S and z_L satisfy $z_S + z_L = \lambda + 1$ is a solution. In this case, $\lambda(1-q)C_1^E + \lambda q C_1^L = z_S\gamma_S R$, and $C_2^L = \frac{(\lambda+1-z_S)}{\lambda q}\gamma_L X$.
2. If $\alpha = 1$, then $z_S = 0$ and $z_L = \lambda+1$. In this case, $C_1^E = C_1^L = 0$, and $\forall C_2^L \leq (\lambda+1)\gamma_L X$ is a solution.

3. If $\alpha < 1$, then $z_S = 0$ and $z_L = \lambda + 1$. In this case, $C_1^E = C_1^L = C_2^L = 0$, $\Pi_1 = 0$, and $\Pi_2 = (\lambda + 1) X$.

The addition of pledgeability constraints alters how much can be promised to consumers out of the produced asset, and may tilt the social planner's preferences over which asset is produced, especially if consumers have high weight. Note that if $\alpha < 1$, the producer's utility matters sufficiently for the planner, so pledgeability does not play a role, and only the long asset is produced. In contrast, if $\alpha > 1$, the consumer's utility matters more for the planner, and the planner weighs the pledgeability adjusted weighted return from each asset in choosing which asset to invest in. For instance, if $\gamma_S = 1$ and $\gamma_L = 0$, the planner will want investment only in the short asset if $\alpha R > X$ because the payoff from the long asset cannot be shared with the consumer.

Pledgeability- and Private Information-Constrained Allocation

When the consumer type is private information, two additional constraints are needed to get types to select the consumption for their type: $C_1^E \geq C_1^L$ to get the early to self select and $C_1^L + C_2^L \geq C_1^E + C_2^E$ for the late. Note that we still have $C_2^E = 0$ because it is always a social waste to offer late consumption to early types and it does not loosen the self selection constraint. The problem becomes

$$
\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha\lambda \left[(1-q)C_1^E + q(C_1^L + C_2^L)\right] + \Pi_1 + \Pi_2 \\
s.t. \; & z_S + z_L = \lambda + 1 \\
& z_S R \geq \Pi_1 \geq z_S(1-\gamma_S)R \\
& z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\
& \lambda(1-q)C_1^E + \lambda q C_1^L \leq z_S \gamma_S R \\
& \lambda q C_2^L \leq z_L \gamma_L X \\
& \lambda(1-q)C_1^E + \lambda q C_1^L + \Pi_1 = z_S R \\
& \lambda q C_2^L + \Pi_2 = z_L X \\
& C_1^E \geq C_1^L \\
& C_1^L + C_2^L \geq C_1^E.
\end{aligned}
$$

Note that the allocations in Lemma 14 satisfy the two constraints. We describe the solution below.

Lemma 15. *In the pledgeability constrained-optimal allocation private information about consumer types does not constrain allocations.*

1. *If* $\alpha > 1$,

   - *If* $\alpha\gamma_S R + (1-\gamma_S) R > \alpha\gamma_L X + (1-\gamma_L) X$, *then* $z_S = \lambda + 1$ *and* $z_L = 0$. *In this case,* $C_1^E = C_1^L = \frac{(\lambda+1)}{\lambda}\gamma_S R$, $C_2^L = 0$, $\Pi_1 = (\lambda+1)(1-\gamma_S) R$, *and* $\Pi_2 = 0$.
   - *If* $\alpha\gamma_S R + (1-\gamma_S) R < \alpha\gamma_L X + (1-\gamma_L) X$, *then* $z_S = 0$ *and* $z_L = \lambda + 1$. *In this case,* $C_1^E = C_1^L = 0$, $C_2^L = \frac{(\lambda+1)\gamma_L X}{\lambda q}$, $\Pi_1 = 0$, *and* $\Pi_2 = (\lambda+1)(1-\gamma_L) X$.
   - *If* $\alpha\gamma_S R + (1-\gamma_S) R = \alpha\gamma_L X + (1-\gamma_L) X$, *then any* z_S *and* z_L *satisfy* $z_S + z_L = \lambda + 1$ *is a solution. In this case, we have* $C_2^L = \frac{(\lambda+1-z_S)}{\lambda q}\gamma_L X$ *and need* $\left\{C_1^E, C_1^L\right\}$ *to satisfy* $\lambda(1-q)C_1^E + \lambda q C_1^L = z_S\gamma_S R$, $C_1^E \geq C_1^L$ *and* $C_1^L + \frac{(\lambda+1-z_S)\gamma_L X}{\lambda q} \geq C_1^E$.[8]

2. *If* $\alpha = 1$, *then* $z_S = 0$ *and* $z_L = \lambda+1$. *In this case,* $C_1^E = C_1^L = 0$, *and* $\forall C_2^L \leq (\lambda+1)\gamma_L X$ *is a solution.*

3. *If* $\alpha < 1$, *then* $z_S = 0$ *and* $z_L = \lambda + 1$. *In this case,* $C_1^E = C_1^L = C_2^L = 0$, $\Pi_1 = 0$, *and* $\Pi_2 = (\lambda+1) X$.

Allocations where producers choose their allocation of investment

When the planner cannot set the allocations z_s and z_L, there is an incentive constraint on producers. Producers obtain all of the non-pledgeable part of any production. That is, only combinations of C_1 and C_2 that are no less profitable than others that the producer could produce are incentive compatible. One way to model this is for consumers to turn over all capital to producers and have them choose z_S and z_L constrained by both competition and producer incentives. We continue to assume that consumers do not trade at date 1. We continue to have $C_2^E = 0$.

The cases of $\alpha = 1$ and $\alpha < 1$ are unchanged. For $\alpha > 1$, we need to compare $\alpha\gamma_S R + (1-\gamma_S) R$ with $\alpha\gamma_L X + (1-\gamma_L) X$. In addition, we need to compare $(1-\gamma_S)R$ with $(1-\gamma_L)X$ to take into account the producers' incentives. Solutions are unchanged if $\alpha\gamma_S R + (1-\gamma_S) R > \alpha\gamma_L X + (1-\gamma_L) X$ and $(1-\gamma_S)R > (1-\gamma_L)X$ or if $\alpha\gamma_S R + (1-\gamma_S) R < \alpha\gamma_L X + (1-\gamma_L) X$ and $(1-\gamma_S)R < (1-\gamma_L)X$, because in both cases, producers' incentives are aligned with the planner's preferences. Two cases remain.

Case 1: $\alpha\gamma_S R + (1-\gamma_S) R > \alpha\gamma_L X + (1-\gamma_L) X$ **and** $(1-\gamma_S)R < (1-\gamma_L)X$. In this case, we need the additional constraint that $\Pi_1 \geq z_S(1-\gamma_L)X$ because when the producers receive z_S, they can instead produce long asset. Let $\lambda(1-q)C_1^E + \lambda q C_1^L = \tilde{C}_1$, and $\lambda q C_2^L = \tilde{C}_2$. The

[8] This set is easily verified to be non-empty.

problem therefore becomes

$$\begin{aligned}\max_{z_S, z_L \in [0,1]} & \alpha \left[\tilde{C}_1 + \tilde{C}_2\right] + \Pi_1 + \Pi_2 \\ s.t.\ & z_S + z_L = \lambda + 1 \\ & z_S R \geq \Pi_1 \geq z_S(1-\gamma_L)X \\ & z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\ & \tilde{C}_1 \leq z_S \gamma_S R \\ & \tilde{C}_2 \leq z_L \gamma_L X \\ & \tilde{C}_1 + \Pi_1 = z_S R \\ & \tilde{C}_2 + \Pi_2 = z_L X.\end{aligned}$$

We further simplify this into

$$\begin{aligned}\max_{z_S, z_L \in [0,1]} & \alpha \left[\tilde{C}_1 + \tilde{C}_2\right] + \left(z_S R - \tilde{C}_1\right) + \left((\lambda + 1 - z_S)X - \tilde{C}_2\right) \\ s.t. & 0 \leq \tilde{C}_1 \leq z_S R - z_S(1-\gamma_L)X \\ & 0 \leq \tilde{C}_2 \leq (\lambda + 1 - z_S)\gamma_L X.\end{aligned}$$

The problem further becomes

$$\begin{aligned}\max_{z_S, z_L \in [0,1]} & (\alpha - 1) \left[\tilde{C}_1 + \tilde{C}_2\right] + z_S (R - X) \\ s.t. & 0 \leq \tilde{C}_1 \leq z_S R - z_S(1-\gamma_L)X \\ & 0 \leq \tilde{C}_2 \leq (\lambda + 1 - z_S)\gamma_L X.\end{aligned}$$

Given that $\alpha > 1$, we have $\tilde{C}_1 = z_S R - z_S(1-\gamma_L)X$ and $\tilde{C}_2 = (\lambda+1-z_S)\gamma_L X$. The objective function becomes

$$(\alpha - 1)(z_S R - z_S(1-\gamma_L)X) + (\alpha - 1)((\lambda + 1 - z_S)\gamma_L X) + z_S (R - X),$$

which is equivalent to

$$\alpha(R - X) z_S.$$

Therefore, it is optimal to let $z_S = 0$ and $z_L = (\lambda + 1)$. In this case, $\tilde{C}_1 = 0$, so that $C_1^E = C_1^L = 0$ and $\tilde{C}_2 = (\lambda + 1)\gamma_L X$ so that $C_2^L = \frac{(\lambda+1)\gamma_L X}{\lambda q}$. It is easily verified that the private information constraints are satisfied.

Case 2: $\alpha\gamma_S R + (1-\gamma_S)\, R < \alpha\gamma_L X + (1-\gamma_L)\, X$ **and** $(1-\gamma_S)R > (1-\gamma_L)X$**.** In this case, we need the additional constraint that $\Pi_2 \geq z_L(1-\gamma_S)R$ because when the producers receive z_L, they can instead produce short asset. Again, let $\lambda(1-q)C_1^E + \lambda q C_1^L = \tilde{C}_1$, and $\lambda q C_2^L = \tilde{C}_2$. The problem therefore becomes

$$\max_{z_S, z_L \in [0,1]} \alpha\left[\tilde{C}_1 + \tilde{C}_2\right] + \Pi_1 + \Pi_2$$

$$\begin{aligned} s.t.\ & z_S + z_L = \lambda + 1 \\ & z_S R \geq \Pi_1 \geq z_S(1-\gamma_L)X \\ & z_L X \geq \Pi_2 \geq z_L(1-\gamma_S)R \\ & \tilde{C}_1 \leq z_S\gamma_S R \\ & \tilde{C}_2 \leq z_L\gamma_L X \\ & \tilde{C}_1 + \Pi_1 = z_S R \\ & \tilde{C}_2 + \Pi_2 = z_L X. \end{aligned}$$

We further simplify this into

$$\max_{z_S, z_L \in [0,1]} \alpha\left[\tilde{C}_1 + \tilde{C}_2\right] + \left((\lambda+1-z_L)R - \tilde{C}_1\right) + \left(z_L X - \tilde{C}_2\right)$$

$$\begin{aligned} s.t. & 0 \leq \tilde{C}_1 \leq (\lambda+1-z_L)\gamma_S R \\ & 0 \leq \tilde{C}_2 \leq z_L X - z_L(1-\gamma_S)R. \end{aligned}$$

Given that $\alpha > 1$, we have $\tilde{C}_1 = (\lambda+1-z_L)\gamma_S R$ and $\tilde{C}_2 = z_L X - z_L(1-\gamma_S)R$. The objective function becomes

$$(\alpha-1)(\lambda+1-z_L)\gamma_S R + (\alpha-1)\left(z_L X - z_L(1-\gamma_S)R\right) + z_L\left(X - R\right),$$

which is equivalent to

$$\alpha z_L\left(X - R\right).$$

Therefore, it is optimal to let $z_S = 0$ and $z_L = (\lambda+1)$. In this case, $\tilde{C}_1 = 0$, so that $C_1^E = C_1^L = 0$ and $\tilde{C}_2 = (\lambda+1)\left[X - (1-\gamma_S)R\right]$ so that $C_2^L = \frac{(\lambda+1)[X-(1-\gamma_S)R]}{\lambda q}$. It is easily verified that the private information constraints are satisfied. Note that we now have $\Pi_2 = (\lambda+1)\,(1-\gamma_S)R$ so that producers receive *more than* the non-pledgeable part of their production.

Proof of Lemma 13

Proof of Lemma 14

Let z_S and z_L be the allocation to short and long-term production at $t = 0$. The problem becomes

$$\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha\lambda \left[(1-q)C_1^E + q(C_1^L + C_2^L)\right] + \Pi_1 + \Pi_2 \\
s.t. \; & z_S + z_L = \lambda + 1 \\
& z_S R \geq \Pi_1 \geq z_S(1-\gamma_S)R \\
& z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\
& \lambda(1-q)C_1^E + \lambda q C_1^L \leq z_S \gamma_S R \\
& \lambda q C_2^L \leq z_L \gamma_L X \\
& \lambda(1-q)C_1^E + \lambda q C_1^L + \Pi_1 = z_S R \\
& \lambda q C_2^L + \Pi_2 = z_L X.
\end{aligned}$$

After the resource constraint, the first four are pledgeability constraints; the last two resource constraints. To solve this problem, let $\lambda(1-q)C_1^E + \lambda q C_1^L = \tilde{C}_1$, and $\lambda q C_2^L = \tilde{C}_2$. We can rewrite the problem as

$$\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha \left[\tilde{C}_1 + \tilde{C}_2\right] + \Pi_1 + \Pi_2 \\
s.t. \; & z_S + z_L = \lambda + 1 \\
& z_S R \geq \Pi_1 \geq z_S(1-\gamma_S)R \\
& z_L X \geq \Pi_2 \geq z_L(1-\gamma_L)X \\
& \tilde{C}_1 \leq z_S \gamma_S R \\
& \tilde{C}_2 \leq z_L \gamma_L X \\
& \tilde{C}_1 + \Pi_1 = z_S R \\
& \tilde{C}_2 + \Pi_2 = z_L X,
\end{aligned}$$

which further becomes

$$\begin{aligned}
\max_{z_S, z_L \in [0,1]} \; & \alpha \left[\tilde{C}_1 + \tilde{C}_2\right] + \left(z_S R - \tilde{C}_1\right) + \left((\lambda + 1 - z_S)X - \tilde{C}_2\right) \\
s.t. \; & 0 \leq \tilde{C}_1 \leq z_S \gamma_S R \\
& 0 \leq \tilde{C}_2 \leq (\lambda + 1 - z_S)\gamma_L X.
\end{aligned}$$

The objective function is equivalent to

$$\left[(\alpha-1)\tilde{C}_1+(\alpha-1)\tilde{C}_2\right]+z_S\,(R-X)$$

The solution is

- If $\alpha>1$, then $\tilde{C}_1=z_S\gamma_S R$ and $\tilde{C}_2=(\lambda+1-z_S)\gamma_L X$, $\Pi_1=z_S\,(1-\gamma_S)\,R$, and $\Pi_2=(\lambda+1-z_S)\,(1-\gamma_L)\,X$. The objective function is equivalent to

  $$[(\alpha-1)(\gamma_S R-\gamma_L X)+(R-X)]\,z_S=\{[\alpha\gamma_S R+(1-\gamma_S)\,R]-[\alpha\gamma_L X+(1-\gamma_L)\,X]\}\,z_S$$

  - If $\alpha\gamma_S R+(1-\gamma_S)\,R>\alpha\gamma_L X+(1-\gamma_L)\,X$, then $z_S=\lambda+1$ and $z_L=0$. In this case, $\lambda(1-q)C_1^E+\lambda qC_1^L=(\lambda+1)\,\gamma_S R$, $C_2^L=0$, $\Pi_1=(\lambda+1)\,(1-\gamma_S)\,R$, and $\Pi_2=0$.
  - If $\alpha\gamma_S R+(1-\gamma_S)\,R<\alpha\gamma_L X+(1-\gamma_L)\,X$, then $z_S=0$ and $z_L=\lambda+1$. In this case, $C_1^E=C_1^L=0$, $C_2^L=\frac{(\lambda+1)\gamma_L X}{\lambda q}$, $\Pi_1=0$, and $\Pi_2=(\lambda+1)\,(1-\gamma_L)\,X$.
  - If $\alpha\gamma_S R+(1-\gamma_S)\,R=\alpha\gamma_L X+(1-\gamma_L)\,X$, then any z_S and z_L satisfy $z_S+z_L=\lambda+1$ is a solution. In this case, $\lambda(1-q)C_1^E+\lambda qC_1^L=z_S\gamma_S R$, and $\lambda qC_2^L=(\lambda+1-z_S)\gamma_L X$

- If $\alpha=1$, then the objective function becomes $z_S(R-X)$ so that $z_S=0$ and $z_L=\lambda+1$. In this case, $C_1^E=C_1^L=0$, and $\forall C_2^L\leq(\lambda+1)\gamma_L X$ is a solution.

- If $\alpha<1$, then $\tilde{C}_1=0$ and $\tilde{C}_2=0$. The objective function becomes

  $$z_S R+(\lambda+1-z_S)X,$$

  in which case, the optimal is always $z_S=0$ and $z_L=\lambda+1$. In this case, $C_1^E=C_1^L=C_2^L=0$, $\Pi_1=0$, and $\Pi_2=(\lambda+1)\,X$.

Proof of Lemma 15

Proof. The proof follows naturally by verifying the allocations in Lemma 14 satisfy the private information constraint. □